MATH 131, Fall 2019   Name: _____
Quiz 10
11/21/19   Section:_____

For full credit you must present a clearly organized solution, showing all supporting calculations.

1. An open top box with a square base is to be made using 12 square feet of cardboard. Find the dimensions that give the largest volume.

2. A dog that can run at 7 meters per second, and can swim at 1.2 meters per second is trying to get to a ball which is floating in the water of a circular pond. The shoreline is 628 meters long, and the ball is 3/4 of the way directly across the lake from the dog who is standing at the shoreline, tilting her head, and calculating the optimal path. At what angle around the pond should the dog enter the water to get to the ball the fastest?

3. Find the dimensions of the rectangle of least perimeter inscribed in the ellipse with equation $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$.

4. Find a pair of positive numbers whose product is 18 and such that the sum of the cube of one number with the cube of twice the other number is a minimum.

5. What is the area of the largest trapezoid that can be inscribed in a radius 1 semi-circle?

6. Find all points on the hyperbola $x^2 - y^2 = 1$ nearest to $(0,3)$. How many are there? How about nearest to $(2,0)$?

7. Show that the points (x,y) on a smooth graph $y = f(x)$ nearest to a given point $P(x_0, y_0)$ not on the graph must lie at the intersection of $y = f(x)$ and *normal lines* through P, which are lines that make right angles with the tangent line to $y = f(x)$ at (x,y).

8. A big cat sanctuary purchased 199,444 square feet of 6 gauge welded square steel wire mesh fencing to complete some new enclosure projects. Given the jumping and climbing abilities of the big cats, the new enclosures must be between 12 and 20 feet high, with roof covering which may be made of the same fencing material as the sides.

(a) The first project is a new safety enclosure connected to a large mountain lion enclosure, where the mountain lions are fed, and can hang out while the main enclosure is cleaned. The safety enclosure will be connected to the existing mountain lion enclosure, and thus only needs three 12 foot tall walls and a roof. Of the total mesh available, 1980 square feet are reserved for this project. Find the dimensions of the enclosure that gives the mountain lions the maximum floor area.

(b) Another project is a new enclosure for a leopard, to be built around a few large trees that the leopard can climb. This enclosure will be 20 feet tall, allowing the leopard to climb up through the sturdier branches of the trees. The enclosure floor will consist of an open rectangular space, bordered by 3 semicircular areas, as in figure 1. Using a calculator, computer algebra system, or a numerical algorithm, find to the nearest square foot the minimum amount of fencing needed if the rectangular portion of the enclosure is to have an area of 2400 square feet.
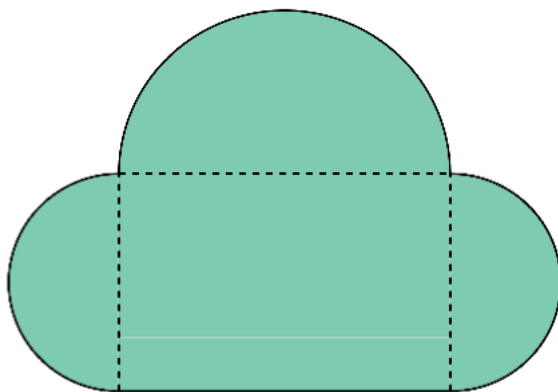

Figure 1

(c) With the remaining fencing, a large rectangular pen with 16 foot tall fencing and roof is to be built to allow ample open space for tigers to wander. While in the wild a tiger's territory may range between 7 and 40 square miles, the sanctuary tries to allocate $1.5\,\mathrm{acres} = 65340$ square feet per tiger. What is the largest number of tigers appropriate for this new enclosure?