MATH 131, Fall 2019
Quiz 9 Solutions

1. Let $f(x) = e^{-x^2}$.

(a) Find the intervals on which f is increasing, and the intervals on which f is decreasing.

$$f'(x) = -2xe^{-x^2} .$$

Observe that $f'(x)$ changes sign precisely once, at $x = 0$ which is the unique critical number. For $x < 0$, $f'(x) > 0$, and for $x > 0$, $f'(x) < 0$ and so $f(x)$ is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.

(b) Find the intervals of concavity and any inflection points of f.

$$f''(x) = (4x^2 - 2)e^{-x^2} = 2(2x^2 - 1)e^{-x^2} .$$

The sign of $f''(x)$ is determined by the sign of the quadratic $2x^2 - 1$, which is positive on $(-\infty, -\sqrt{2}/2) \cup (\sqrt{2}/2, \infty)$ and negative on $(-\sqrt{2}/2, \sqrt{2}/2)$. Thus $f(x)$ is concave up on $(-\infty, -\sqrt{2}/2) \cup (\sqrt{2}/2, \infty)$ and concave down on $(-\sqrt{2}/2, \sqrt{2}/2)$, and the points of inflection are $(-\sqrt{2}/2, 1/\sqrt{e})$ and $(\sqrt{2}/2, 1/\sqrt{e})$.

(c) Find and classify all local extrema using either the first or second derivative tests.

At the unique critical number $x = 0$ the derivative is changing from positive to negative, whence $(0, 1)$ is a local maximum.

(d) Determine any asymptotes of f.

Note that $f(x)$ is continuous for all real x and thus there are no vertical asymptotes. However,

$$\lim_{x\to-\infty} e^{-x^2} = 0 \lim_{x\to\infty} e^{-x^2} = 0 ,$$

whence $y = 0$ is a horizontal asymptote of $y = e^{-x^2}$.

(e) Use the information gathered in (a)-(d) to sketch a graph of f.

See figure 1.

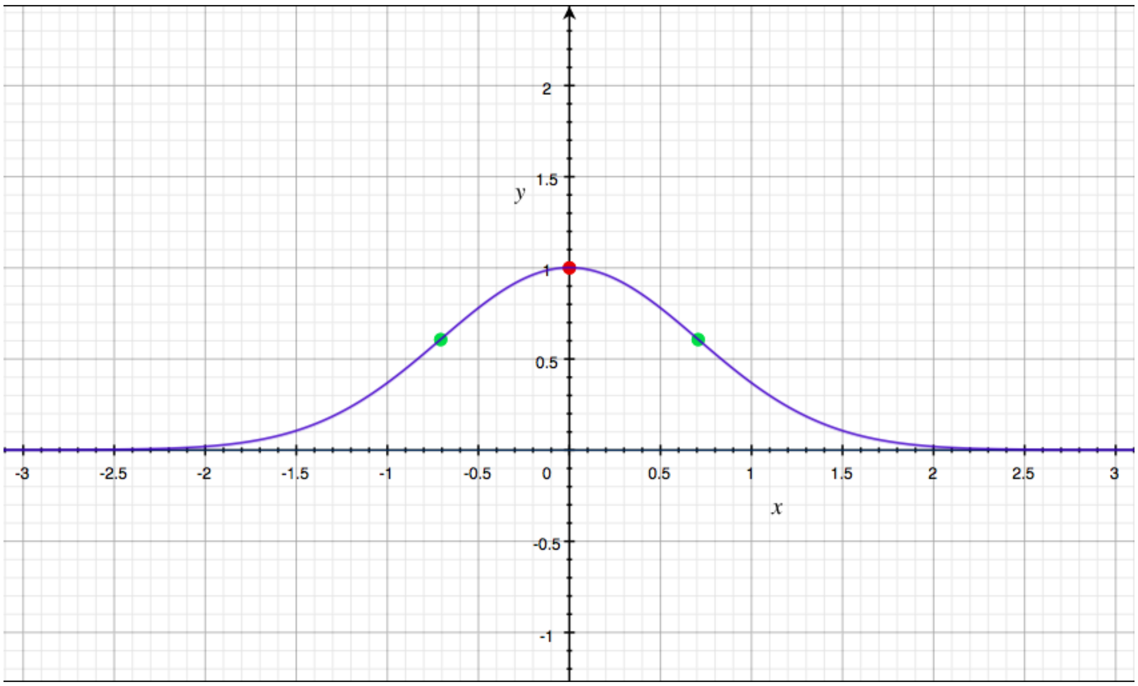

Figure 1: The graph of $y = e^{-x^2}$.