MATH 131, Fall 2019 Name: _____
Quiz 8
11/07/19 Section:_____

For full credit you must present a clearly organized solution, showing all supporting calculations. This quiz has two sides.

1. Show that for any real numbers a and b,

$$|\cos b - \cos a| \leq |b - a| .$$

2. Show that $f(x) = 3x - e^{\cos x}$ has a unique real root.

3. Show that for all $x \geq 1$ the functions $f(x) = 2\cot^{-1}\left(\frac{1}{\sqrt{x}}\right)$ and $g(x) = \cos^{-1}\left(\frac{2\sqrt{x}}{1+x}\right)$ differ by a constant, and determine the value of that constant.

4. A point $x = a$ is called a *fixed point*[1] of a function $f(x)$ if $f(a) = a$. Show that if $f'(x) \neq 1$ for all x then f has at most one fixed point.

[1]The terminology "fixed point" is evocative if one considers the effect of *iterating* the map $x \mapsto f(x)$: if f is a map from a domain $D \subseteq \mathbb{R}$ with range contained in D, then consider for any number c in the domain D of f the sequence $\mathcal{O}(f, c) := \left\{c, f(c), f\big(f(c)\big), f\Big(f\big(f(c)\big)\Big), \ldots, f^n(c), \ldots\right\}$, called the *orbit of c under the map $x \mapsto f(x)$* . Then a fixed point $x = a$ is one for which this sequence is constant: repeated applications of f leave a fixed, and a has *constant orbit'* under f.