MATH 131, Fall 2019
Quiz 4 Solutions

1. Calculate the following derivatives by any means necessary. You do not need to simplify your final answers.

(a) $f'(x)$ for $f(x) = \dfrac{x^2 - 2\sqrt{x} + 1}{x^3}$,

(b) $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\Big((2^\theta - \theta^2)\cot(\theta)\Big)$,

(c) $g''(t)$ for $g(t) = \sec t$.

(a) Simplifying the function first, then applying the power rule:

$$f(x) = \frac{x^2 - 2\sqrt{x} + 1}{x^3} = x^{-1} - 2x^{-5/2} + x^{-3},$$

$$f'(x) = -x^{-2} + \frac{5}{2}x^{-7/2} - 3x^{-4}.$$

(b) Using the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\Big((2^\theta - \theta^2)\cot(\theta)\Big) = (2^\theta \ln(\theta) - 2\theta)\cot(\theta) - (2^\theta - \theta^2)\csc^2(\theta)$$

(c)

$$g'(t) = \sec t \tan t.$$

$$g''(t) = \sec t \tan^2 t + \sec^3 t.$$

2. Let $f(x) = e^x \cos x$. Find $f^{(18)}(x)$, the 18th derivative of $f(x)$.

By taking a few derivatives we can observe a pattern:

$$\begin{aligned}
f'(x) &= e^x \cos x - e^x \sin x, \\
f''(x) &= e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x = -2e^x \sin x, \\
f'''(x) &= -2e^x \sin x - 2e^x \cos x = f''(x) - 2f(x), \\
f^{(4)}(x) &= f'''(x) - 2f'(x) = -4e^x \cos x = -4f(x).
\end{aligned}$$

Thus, for any integer $k \geq 0$:

$$f^{(4k)}(x) = (-4)^k f(x),$$

whence

$$f^{(18)}(x) = f^{(2+4\cdot 4)}(x) = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left((-4)^4 e^x \cos x\right) = 256 f''(x) = -512 e^x \sin x\,.$$