MATH 131, Fall 2019
Quiz 2 Solutions

1. Use the limit laws to evaluate
$$\lim_{x\to 2} \ln\left[\frac{x^2-2x}{x^3-8}\right].$$

Note that the fraction $(x^2-2x)/(x^3-8)$ is positive for $x>0$. By continuity of $\ln x$ on its domain $(0,\infty)$, we can pass the limit inside and simplify the fraction within the limit:

$$\begin{aligned}\lim_{x\to 2} \ln\left[\frac{x^2-2x}{x^3-8}\right] &= \ln\left[\lim_{x\to 2}\frac{x^2-2x}{x^3-8}\right]\\ &= \ln\left[\lim_{x\to 2}\frac{x(x-2)}{(x-2)(x^2+2x+4)}\right]\\ &= \ln\left[\lim_{x\to 2}\frac{x}{(x^2+2x+4)}\right]\\ &= \ln\left[\frac{2}{(2^2+2(2)+4)}\right]\\ &= \ln\left[\frac{2}{12}\right] = -\ln 6\,.\end{aligned}$$

2. Find all numbers c such that the function
$$f(x) = \begin{cases} 9-x^2 & \text{if } x \geq 3 \\ -x^3-7cx-4c^2 & \text{if } x < 3 \end{cases}$$
is continuous on $(-\infty,\infty)$.

Note that $f(x)$ is piecewise and the pieces are each polynomials in x, and thus the pieces are continuous over the subdomains in which they are used (because, as functions on $\mathbb{R}$ they are continuous throughout $\mathbb{R}$). To determine c we need to ensure the equality of left and right limits at the transition between pieces, that is, we need

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$$

Observe that by restricting the domain and using direct substitution

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(9-x^2) = 9-(3)^2 = 0\,,$$

thus, we must choose c so that

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(-x^3-7cx-4c^2) = 0\,.$$

By direct substitution again, we obtain the equation

$$-27 - 21c - 4c^2 = 0\,.$$

This factors as $-(c+3)(4c+9) = 0$. Thus c could either be equal to -3 or $-9/4$.