MATH 131, Fall 2019
Quiz 2 Solutions

1. Evaluate the limit

$$\lim_{x\to -4} \sqrt{x^2+9}\,\cos\left(\pi\left(\frac{4x^2+16x}{16-x^2}\right)\right),$$

using the limit laws. Show all steps of simplifying within limits clearly. If you apply the direct substitution property, you should indicate where it is applied and justify the application.

Let

$$f(x) = \sqrt{x^2+9}\cos\left(\pi\left(\frac{4x^2+16x}{16-x^2}\right)\right),$$

and observe that $f(x) = g(x)\cos\big(h(x)\big)$ where $g(x) = \sqrt{x^2+9}$ is algebraic, and $h(x) = \pi(4x^2+16x)/(16-x^2)$ is rational. We'll split the product and apply limits to each factor. Since cosine is continuous, we can move a limit operator inside to evaluate the limit of the rational function $h(x)$, once it is simplified, by direct substitution. Thus

$$\begin{aligned}
\lim_{x\to -4} f(x) &= \left(\lim_{x\to -4}\sqrt{x^2+9}\right)\left(\lim_{x\to 4}\cos\left(\pi\left(\frac{4x^2+16x}{16-x^2}\right)\right)\right) && \text{(Limit of a product)}\\
&= \left(\sqrt{\lim_{x\to -4}(x^2+9)}\right)\lim_{x\to -4}\cos\left(\pi\left(\frac{4x^2+16x}{16-x^2}\right)\right) && \text{(Limit of a Positive Root)}\\
&= \left(\sqrt{4^2+9}\right)\lim_{x\to -4}\cos\left(\pi\left(\frac{4x^2+16x}{16-x^2}\right)\right) && \text{(Direct Substiution)}\\
&= 5\cos\left(\pi\lim_{x\to -4}\left(\frac{4x^2+16x}{16-x^2}\right)\right) && \text{(Continuous Composition and Linearity)}\\
&= 5\cos\left(\pi\lim_{x\to -4}\left(\frac{4x(x+4)}{(x+4)(4-x)}\right)\right) && \text{(Factoring)}\\
&= 5\cos\left(\pi\lim_{x\to -4}(4x)\lim_{x\to -4}\left(\frac{x+4}{(x+4)(4-x)}\right)\right) && \text{(Limit of a Product)}\\
&= 5\cos\left(\pi(-16)\lim_{x\to -4}\left(\frac{1}{4-x}\right)\right) && \text{(Direct Substitution and Simplifying)}\\
&= 5\cos\left(-16\pi\left(\frac{1}{8}\right)\right) && \text{(Direct Substitution)}\\
&= 5\cos(-2\pi) && \text{(Simplifying)}\\
&= 5\,. && \text{(Evaluation)}
\end{aligned}$$