Problem 8. It’s possible to use non-complex analysis to solve this problem, but what is the fun in that?

I found it more natural to work with constants $A > B > 0$ satisfying

$$A^2 + B^2 = a, \quad -2AB = b.$$ (Think about the existence and uniqueness of $A, B$). Note that this change of variables “rationalizes” the value of the integral (gets rid of the square root).

At one point, I found it more convenient to go to just one quantity $K = A/B > 1$. You may wish to begin by evaluating

$$\int_{C_A(0)} \frac{1}{z + B} dz$$

directly as well as by the Residue Theorem. At a later point, you may want to separate the integral into its real and imaginary components. Don’t be sheepish about using some computer algebra package to help you come up with complex analysis ideas for cracking the problem.

Problem 14. Use theorem 3.4.

Problem 15 (d) The fact that the real part is bounded makes the image land in what kind of “strip?” Think of a wonderful, useful, function that we use all the time, call it $W(z)$ and consider the function $g(z) = W(f(z))$. [What you are looking for is a function $W$ that does something nice to “vertical strips.”]. Who is the usual suspect when the words “entire and bounded” are uttered?

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Warning: There may very well be [probably are] more efficient roads to solving the problems than the roads suggested here.