I. Stein-Shakarchi Chapter 1, problems 1,2,3,5,7.

II. (a) Prove that the points $z_1, z_2, z_3$ in the complex plane are vertices of an equilateral triangle if and only if

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_1z_3 + z_2z_3.$$  

(b) Discover and demonstrate a condition involving the ratio \((z_1 - z_3)/(z_2 - z_3)\) for three distinct points $z_1, z_2, z_3$ to be collinear.

(c) The cross-ratio of a sequence of four distinct points $z_1, \ldots, z_4$ in the complex plane is defined to be

$$\frac{(z_1 - z_2)/(z_2 - z_3)}{(z_1 - z_4)/(z_2 - z_4)}.$$  

Discover and demonstrate a condition involving the cross-ratio for the property that the points $z_1, \ldots, z_4$ are either collinear or lie on a common circle.

III. (a) Show that for $A \neq 0$, the set of all points $(x, y)$ in $\mathbb{R}^2$ satisfying $Ax^2 + Ay^2 + Bx + Cy + D = 0$ is either empty or a circle. Determine the center and the radius. What happens when $A = 0$?

(b) Suppose $K$ is a positive real number, $K \neq 1$. Show that the set of all $z \in \mathbb{C}$ satisfying

$$\frac{|z - z_1|}{|z - z_2|} = K$$

is a circle. What is its radius? What happens when $K = 1$?

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For the due date, check the course web page.