

# UMASS AMHERST MATH 455 F. HAJIR

## EXAM 1 SAMPLE

Exam 1 will take place Thursday April 11 in class. Don't be late. For the true/false and short answer sections, you do not need to justify your answer, just GET the answer. For the "proofs" part, you DO need to justify your steps. Don't spend too much time on the short answer/definitions/TF. You want to leave enough time to think about the proofs which carry half the points roughly. I have included more problems in this sample exam than there will be on the actual exam (in order to give you more practice).

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1. Define the following terms. Unless otherwise stated,  $G$  denotes a SIMPLE graph with vertex set  $V$  of size  $n$  and edge set  $E$  of size  $q$ .

- a. We say  $G$  has an Eulerian trail if
- b. A  $k$ -coloring of  $G$  is
- c. The chromatic number of  $G$  is
- d. We say  $G$  is planar if
- e. Describe the steps of the Kruskal algorithm.
- f. If  $S$  is a subset of  $V$ , the subgraph of  $G$  induced by  $S$  is
- g. The chromatic polynomial  $c_G(k)$  of  $G$  is defined by
- h. A graph is called acyclic if
- i. The theorem of Cayley on enumeration of trees states that

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2. True/False

- a. All trees of order  $n$  have the same chromatic polynomial.
- b. If  $G$  is a graph of order  $n$  satisfying  $\chi(G) = n$ , then  $G$  is isomorphic to  $K_n$ .
- c. If every edge of a graph  $G$  lies on an odd number of cycles, then  $G$  is Eulerian.
- d. Every Eulerian graph is Hamiltonian.

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3. Short Answer

a. If  $G$  is a connected planar graph of order 24, and it is regular of degree 3, how many regions are there in a planar representation of  $G$ ?

b. Compute the chromatic polynomial of the square (4-cycle).

c. Find a 4-regular planar graph.

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4. Proofs

a. Prove that there is no bipartite planar graph with  $\delta(G) \geq 4$ .

b. Show that  $k^4 - 4k^3 + 3k^2$  is not the chromatic polynomial of any graph.

c. Prove that if  $G$  is Hamiltonian, then it is 2-connected.

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