

# UMASS AMHERST MATH 455 F. HAJIR

## EXAM 1 SAMPLE

Exam 1 will take place Tues February 12 in class. Don't be late. For the true/false and short answer sections, you do not need to justify your answer, just GET the answer. For the "proofs" part, you DO need to justify your steps. Don't spend too much time on the short answer/definitions/TF. You want to leave enough time to think about the proofs which carry half the points roughly. I have included more problems in this sample exam than there will be on the actual exam (in order to give you more practice).

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1. Define the following terms. Unless otherwise stated,  $G$  denotes a SIMPLE graph with vertex set  $V$  of size  $n$  and edge set  $E$  of size  $m$ .

- a. The degree of a vertex  $v$  is defined to be
- b. The distance from a vertex  $u$  to a vertex  $v$  is defined by  $\text{dist}(u, v) =$
- c. The eccentricity of a vertex  $v$  is defined by  $\text{ecc}(v) =$
- d. A graph  $G$  is called bipartite means that
- e. A walk (path, trail, cycle) is
- f. The First Theorem of Graph Theory (or the Handshake Theorem) states that
- g. A graph  $G$  is called  $k$ -connected if
- h. If  $G$  has vertices  $v_1, \dots, v_n$ , then the adjacency matrix with respect to this ordering of the vertices is defined by
- i. The complement  $\overline{G}$  of  $G$  is defined by

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2. True/False

- a. If a simple graph on  $n$  vertices is not complete, then there are at least two vertices of degree at most  $n - 2$ .
- b. If  $G$  is a connected graph with at least three vertices, then for every vertex  $v$  of  $G$  we have  $\deg(v) \geq 2$ .
- c. If  $H = L(G)$  is the line graph of a simple graph  $G$ , then  $L(H)$  is isomorphic to  $H$ .

d. For every graph  $G$ , the center of  $G$  has at least two elements.

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3. Short Answer

a. Draw a picture of  $K_{3,3}$ . Choose an ordering of vertices of the complete bipartite graph  $K_{3,3}$  and compute the corresponding adjacency matrix. Be sure to label the vertices in your picture of the graph.

b. Find a graph of order 7 that has radius 3 and diameter 6.

c. Draw the line graph  $H = L(G)$  of the complete bipartite graph  $G = K_{2,3}$ . How many edges does  $H$  have?

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4. Proofs

a. Prove that an edge  $e$  of  $G$  is a bridge if and only if  $e$  lies on no cycle of  $G$ .

b. Prove that if  $A$  is the adjacency matrix of  $G$  with vertices ordered as  $v_1, \dots, v_n$ , then for each  $1 \leq j \leq n$ , the degree of  $v_j$  is the  $j$ th row,  $j$ th column entry of the matrix  $A^2$ .

c. Prove that in every 2-connected graph, there is at least one cycle.

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