1. Reading

Please read 6.1, 6.3, 6.4, 6.5, and 6.6 of Gilbert/Vanstone. You should also read Part III of Farshid’s notes.

2. Problems from Gilbert/Vanstone

None.

3. Problems from Farshid’s Brain

1. Write down the negation of the following statements; in each case attempt to cast the statement in positive terms, meaning attempt to eliminate the word “not” from your statement. For example, while it is true that the negation of $P$: The integer $n$ is odd is $\neg P$: The integer $n$ is not odd, it is more useful to write $\neg P$: The integer $n$ is even.

(a) $A$: The triangle $ABC$ is equilateral.
(b) $B$: For all real numbers $x \geq 0$, $x^2 - x \leq 0$.
(c) $C$: There exist integers $m, n$ such that $m^2 = 2n^2$.
(d) $D$: If $a$ and $b$ are integers with $\gcd(a, b) = 2$, then there exist integers $x$ and $y$ such that $ax + by = 1$.

2. In this problem, let $a, b, c$ be integers. Write down the converse of the following statements:

(a) If $a$ and $b$ satisfy $\gcd(a, b) = 1$, then there exist integers $x, y$ such that $ax + by = 1$.
(b) If $a$ divides $bc$, then either $a$ divides $b$ or $a$ divides $c$.
(c) Show that (b) is false by providing a counterexample.

3. Suppose $a, b, c$ denote the lengths of the three edges of some triangle in the plane. Write down first the converse and then the contrapositive of the following statement.

$R$: If the angle subtended by the sides of length $a$ and $b$ is 90 degrees, then $a^2 + b^2 = c^2$.

Give your opinion on the validity of $R$, its converse and its contrapositive.

4. Consider the following sets

$$A = \{x \in \mathbb{R} | x^2 - x \leq 0\}$$
$$B = \{x \in \mathbb{R} | -(x - 1)(x - 3) \leq 0\}$$
$$C = \{x \in \mathbb{R} | x \geq 1\}.$$ 

(a) Determine $A \cap B$.
(b) Determine $A \cap C$.