1. Reading

You should read all of Chapter 1 in Gilbert/Vanstone as well as Part II of Farshid’s class notes.

2. Problems from Gilbert/Vanstone Chapter 1

Exercise Set 1 (p. 20): 1-13, 25, 41-44, 56, 58, 60, 64, 66, 69.
Problem Set 1 (p. 22): 74, 76.

3. Problems from Farshid’s brain

1. Prove that \( P \implies (P \lor Q) \) is a tautology, i.e. its truth table has value “True” in all cases.

2. Consider the following statement.
   \( A \): All residents of Amherst MA are residents of Massachusetts.
   (a) Rewrite \( A \) in the form of an implication i.e. in the form If \( P \), then \( Q \).
   (b) Now give the converse of \( A \).
   (c) Is \( A \) true? Explain.
   (d) Is the converse of \( A \) true? Explain.

3. (a) Suppose the union of ten sets \( A_1 \cup A_2 \cup \cdots \cup A_{10} \) equals \( A_1 \). What can you conclude about these sets?
   (b) Suppose the intersection of ten sets \( B_1 \cap B_2 \cap \cdots \cap B_{10} \) is \( B_1 \). What can you conclude about these sets?

4. Consider the sets \( A = \{0, 1\}, B = \{a, b, c\} \). List the elements of the sets \( A \times A, A \times B, B \times A, A \times B \times A \).

5. Prove that if \( P \implies Q \) and \( Q \implies R \) and \( R \implies P \), then \( P, Q, R \) are all pairwise equivalent.

4. Extra Credit

1. Construct a sequence of sets \( S_1, S_2, S_3, \ldots \) (one for each natural number) such that for any finite subset \( \{i_1, \ldots, i_n\} \subseteq \mathbb{N} \) of the natural numbers, the intersection \( S_{i_1} \cap \cdots \cap S_{i_n} \) is an infinite set, but \( \cap_{n \geq 1} S_n = \{\} \).