1. Write the following system of equations as a matrix equation and find all solutions using Gaussian elimination:

\[
\begin{align*}
  x + 2y + 4z &= 0, \\
  -x + 3y + z &= -5, \\
  2x + y + 5z &= 3.
\end{align*}
\]

2. (a) Suppose \( T \) is a linear transformation represented by a matrix \( A \). What does it mean for a vector \( v \) to be in the kernel of \( T \) (or, equivalently, in the kernel of the matrix \( A \))?

(b) Let \( A \) be the matrix \[
\begin{pmatrix}
  1 & 2 & 5 \\
  -2 & 0 & -2 \\
  3 & -1 & 1
\end{pmatrix}
\]. Is \[
\begin{pmatrix}
  1 \\
  2 \\
  -1
\end{pmatrix}
\] an element of the kernel of \( A \)? Why or why not?

3. Let \( V \subseteq \mathbb{R}^n \) be a subspace of \( \mathbb{R}^n \). Define what it means for a list \( v_1, \ldots, v_k \) of vectors in \( \mathbb{R}^n \) to be a) linearly independent, b) to span \( V \), and c) to be a basis of \( V \).

Let \[
A = \begin{pmatrix}
  1 & 2 & 3 & -1 \\
  -1 & 0 & 1 & -1 \\
  -1 & 4 & 3 & -5
\end{pmatrix}.
\]

Give a list of linearly independent vectors that span \( \ker(A) \).

4. Let \( A \) be a \( n \) by \( m \) matrix, so \( A \) gives a linear transformation from \( \mathbb{R}^m \) to \( \mathbb{R}^n \). Let \( x_1, x_2 \in \mathbb{R}^m \). Assume that \( A(x_1) = A(x_2) \). Show that \( x_1 - x_2 \) is in the kernel of \( A \). What property of matrix multiplication did you use in the process?

5. Let \( L \) be a line through the origin and let \( u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \) be a vector of length 1 lying along that line. Let \( A \) be a matrix whose effect on the plane is to reflect about the line \( L \). Let \( v = \begin{pmatrix} -u_2 \\ u_1 \end{pmatrix} \). In terms of \( u \) and \( v \) what is \( A(u) \)? what is \( A(v) \)? Write \( e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) as a linear combination of \( u \) and \( v \). Use the answer to the previous question to compute \( A(e_1) \).

6. Let

\[
A = \begin{pmatrix}
  1 & 0 & -1 \\
  0 & 1 & 2 \\
  2 & 1 & -1
\end{pmatrix}.
\]

(a) Determine whether the columns of \( A \) are linearly independent or not. Show your work.

(a) Use the row reduction method to find the inverse matrix \( A^{-1} \).
(b) By using $A^{-1}$, solve the system

$$A\vec{x} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

for $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

7. Compute the product $AB$ of the two matrices $A, B$ given below, if possible. If it is not possible say why it is not possible.

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & -2 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 0 \\ 4 & 8 \end{pmatrix}$$

The product matrix $AB$ represents a linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$. Determine $m$ and $n$.

8. Find a basis of the subspace of $\mathbb{R}^3$ defined by $3x - y + z = 0$. What is the dimension of this subspace?

9. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 1 & 10 \\ 3 & 1 & c \end{pmatrix}.$$  

a) Find all the value(s) of $c$ for which the matrix $A$ is not invertible.

b) For what value(s) of $c$ does the linear system

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ 20 \end{pmatrix}$$

have no solution?

10. Suppose $A$ and $B$ are $n \times n$ matrices such that $AB = 0$ where $0$ is the $n \times n$ all-zeros matrix. Show that if $A$ is invertible, then $B = 0$. 

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