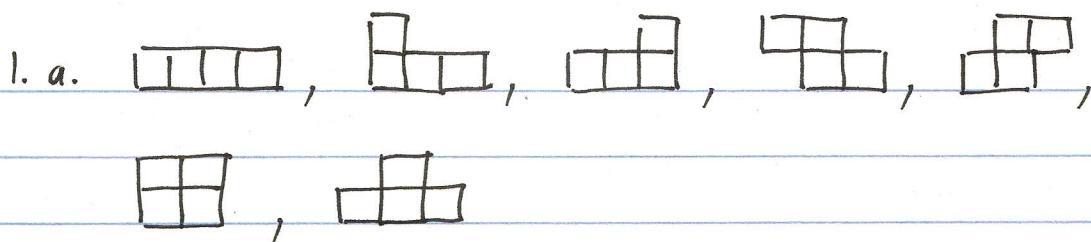




Saturday 11/7/15.

Teaching Seminar worksheet solutions.

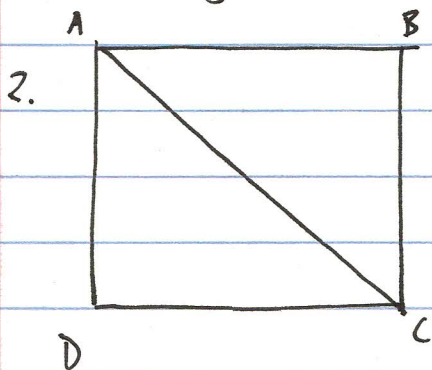


b. Divide the 4×7 rectangle into 1×1 squares, & colour the squares alternately black & white (like a chessboard).

Notice that all the tiles except the last one () cover ~~2~~ 2 black squares & 2 white squares, whereas  will cover 3 black & 1 white or vice versa.

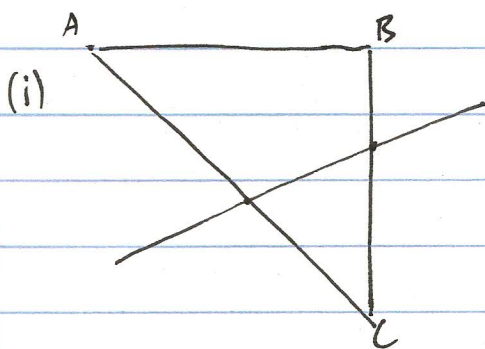
But the 4×7 rectangle has the same number of black & white squares (14 of each).

This shows that it is not possible to tile the rectangle using the 7 tetrominos.

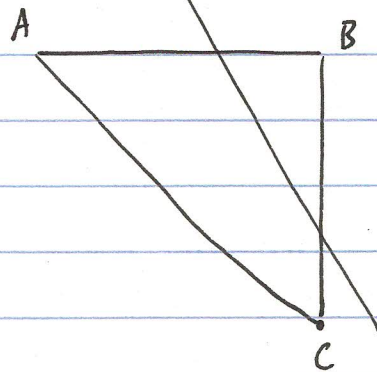


(up to the symmetry switching A & C)

There are two possible types of cut:

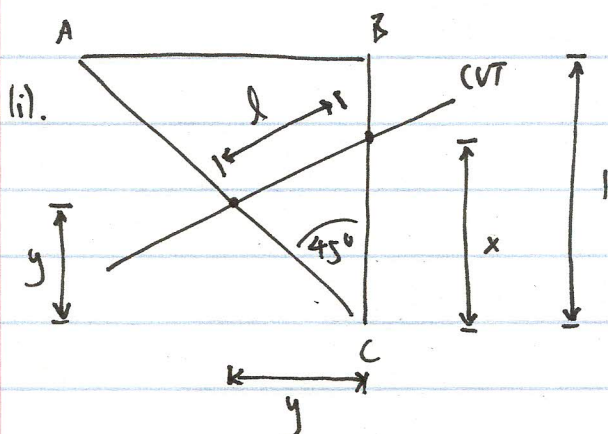


(ii)



Draw a diagram & introduce notation.

We may assume (by scaling)
square ABCD has side length 1.



Condition: Two parts have equal area.

$$\text{Area of lower part} = \frac{1}{2}xy = \frac{1}{2} \text{area}(\triangle ABC) = \frac{1}{4}$$

i.e., require $xy = \frac{1}{2}$.

Goal: Minimize length l of cut.

$$l = \sqrt{y^2 + (x-y)^2} = \sqrt{x^2 - 2xy + 2y^2} \quad \dagger$$

Equivalently, minimize $z := l^2 = x^2 - 2xy + 2y^2$
subject to $xy = \frac{1}{2}$.

Eliminate y : $y = \frac{1}{2x}$ $\dagger\dagger \Rightarrow z = x^2 - 1 + \frac{1}{2x^2}$

Find critical points: $\frac{dz}{dx} = 2x - \frac{2}{2x^3} = 2x - \frac{1}{x^3}$

$$\frac{dz}{dx} = 0 \Leftrightarrow x^4 = \frac{1}{2}, \quad x = \frac{1}{2^{1/4}} \quad (x > 0)$$

