235 Midterm 2 review questions

November 9, 2016

(1) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 5 \\ 4 & 1 & 1 \end{bmatrix}$.

(a) Compute the determinant of A.

(b) Is A invertible?

(2) (a) Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

(b) What is the determinant of the matrix obtained by interchanging row 1 and row 2 of A? Justify your answer.

(c) What is the determinant of the matrix obtained by interchanging row 2 and row 3 of A? Justify your answer.

(d) What is the determinant of the matrix obtained by multiplying row 3 of A by the scalar 10? Justify your answer.

(e) What is the determinant of the matrix obtained by replacing row 1 of A with (row 1) − 47(row 2)? Justify your answer.

(3) (a) Compute the volume of the parallelepiped S in $\mathbb{R}^3$ which has one vertex at the origin and adjacent vertices at $(2,0,1)$, $(0,1,1)$ and $(1,3,2)$.

(b) Let T_A be the linear transformation of $\mathbb{R}^3$ with standard matrix

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 3 & 7 \\ 0 & 0 & -1 \end{bmatrix}$$

Using your answer to part (a) or otherwise, find the volume of the image under T_A of the parallelepiped S.

(4) Justify your answers carefully.

(a) Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x, y \text{ in } \mathbb{R} \text{ and } y = x^2 \right\}$. Is H a subspace of $\mathbb{R}^2$?

(b) Let $\mathbb{P}_2$ be the vector space of polynomials in the variable t of degree ≤ 2. Let $K = \left\{ at^2 + bt + c \mid a, b, c \text{ in } \mathbb{R} \text{ and } a + b = 1 \right\}$. Is K a subspace of $\mathbb{P}_2$?

(5) For each of the following functions, determine if they are linear transformations

(a) $T : \mathbb{P}_3 \to \mathbb{R}^2$ given by $T\left(\mathbf{p}(t)\right) = \begin{bmatrix} \mathbf{p}(1) \\ \mathbf{p}(2) \end{bmatrix}$.

(b) $U : \mathbb{R}^2 \to \mathbb{P}_2$ given by $U\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = a^2 t + b$.

(6) Let $H = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; x, y, z \text{ in } \mathbb{R} \text{ and } x + 2y + 7z = 0 \right\}$.

(a) Show that H is a subspace of $\mathbb{R}^3$.

(b) Find a basis of H.

(c) What is the dimension of H?

(7) Let $H = \left\{ \begin{bmatrix} a + b + 3c \\ a + 2c + 2d \\ b + c \\ a + 2c + d \end{bmatrix} \;\middle|\; a, b, c, d \text{ in } \mathbb{R} \right\} \subset \mathbb{R}^4$.

(a) Show that H is a subspace of $\mathbb{R}^4$.

(b) Find a basis of H.

(c) What is the dimension of H?

(8) Let $A = \begin{bmatrix} 1 & 3 & 2 & 7 & 2 \\ 1 & 3 & 1 & 2 & 3 \\ 2 & 6 & 3 & 9 & 5 \end{bmatrix}$.

(a) Find a basis of the null space of A.

(b) Find a basis of the column space of A.

(9) Let

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}.$$

(a) Show that $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis of $\mathbb{R}_3$.

(b) Find the vector $\mathbf{v}$ in $\mathbb{R}^3$ with $\mathcal{B}$-coordinate vector $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

(c) Find the $\mathcal{B}$-coordinate vector $[\mathbf{w}]_{\mathcal{B}}$ of the vector $\mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$.

(10) Let $\mathbb{P}_2$ be the vector space of polynomials in the variable t of degree ≤ 2. Let $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ be the following polynomials in $\mathbb{P}_2$:

$$\mathbf{b}_1(t) = 1 + 2t + t^2, \quad \mathbf{b}_2(t) = 1 + 3t + 4t^2, \quad \mathbf{b}_3(t) = 1 + 4t + 8t^2.$$

(a) Show that $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis of $\mathbb{P}_2$.

(b) Let $\mathbf{p}$ be the polynomial $\mathbf{p}(t) = t^2$ in $\mathbb{P}_2$. Find the $\mathcal{B}$-coordinate vector $[\mathbf{p}]_{\mathcal{B}}$ of $\mathbf{p}$.