1. Find all solutions of the following systems of linear equations. Explain your results geometrically.

(a) 
\begin{align*}
  x + 2y + 3z &= 5 \\
  2x + y + 2z &= 1 \\
  x - y + 3z &= 2 
\end{align*}

(b) 
\begin{align*}
  x + y + z + t &= 2 \\
  x + 2y + 3z + 2t &= 4 \\
  2x + 4y + 4z - t &= 5 
\end{align*}

2. Find all solutions of the vector equation 
\[ x_1 v_1 + x_2 v_2 + x_3 v_3 = b \]
where 
\[ v_1 = \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, b = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}. \]
Explain your result geometrically.

3. Find all solutions of the equation \( Ax = b \) where 
\[ A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & -1 & 0 \\ 1 & 2 & 1 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}. \]
Does the equation $Ax = c$ have a solution for every vector $c$ in $\mathbb{R}^3$? Explain your answer.

4. Let $S$, $T$, $U$, $V$ be the linear maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by the matrices

\[
\begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
-1 & 1
\end{pmatrix}, \quad \begin{pmatrix}
0 & -1 \\
-1 & 0
\end{pmatrix}, \quad \begin{pmatrix}
1 & 0 \\
2 & 1
\end{pmatrix}.
\]

(a) Describe the maps geometrically. (It may help to draw the image of the unit square.)

(b) Compute the matrices of the compositions $S \circ U$, $T \circ T$, and $T^{-1} \circ U \circ T$. Interpret your results geometrically.

5. Find the matrix of the following linear maps.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotation about the origin through an angle of $\pi/3$ radians anticlockwise.

(b) $U : \mathbb{R}^2 \to \mathbb{R}^2$ reflection in the line through the origin in direction $\left(\frac{1}{2}\right)$.

(c) $V : \mathbb{R}^3 \to \mathbb{R}^3$ projection onto the plane $x + 2y + 3z = 0$.

(d) $W : \mathbb{R}^3 \to \mathbb{R}^3$ rotation about the $y$-axis through an angle of $\pi/3$ radians anticlockwise.

6. 

(a) The linear map $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $A = \frac{1}{13} \begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}$ is a projection onto a line. Find the line.

(b) The linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by the matrix $B = \frac{1}{3} \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$ is a reflection in a plane. Find the plane.

(c) The linear map $U : \mathbb{R}^3 \to \mathbb{R}^3$ given by the matrix $C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$ is a rotation. Find the axis of rotation.

7. Let

\[
A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ -1 & -1 & 2 \end{pmatrix}.
\]
(a) Compute $A^{-1}$.

(b) Using your result from (a), solve the linear system

\[
\begin{align*}
    x_1 + 2x_2 + x_3 &= 1 \\
    2x_1 + 4x_2 + 3x_3 &= 0 \\
    -x_1 - x_2 + 2x_3 &= 0
\end{align*}
\]

8. The unit cube in $\mathbb{R}^3$ has vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(1,1,0)$, $(1,0,1)$, $(0,1,1)$, $(1,1,1)$. Suppose $T: \mathbb{R}^3 \to \mathbb{R}^2$ is a linear map such that $T(1,0,0) = (2,1)$, $T(0,1,0) = (1,2)$, and $T(0,0,1) = (1,1)$. Write down the matrix of $T$. Draw the image of the unit cube in $\mathbb{R}^2$ under the map $T$ (draw the image of each edge of the cube).