

Name: .....

ID Number: .....

Section Number: .....

Section	Instructor	Day/Time	Section	Instructor	Day/Time
1	Farelli	MWF 10:10	9	Benincasa	TuThu 1:00
2	Farelli	MWF 9:05	10	Benincasa	TuThu 2:30
3	Clark	MWF 11:15	11	Buskin	MWF 10:10
4	Clark	MWF 12:20	12	Yaping	MWF 12:20
5	Brown	MW 2:30	13	Yaping	MWF 1:25
6	Brown	MW 4:00	15	Buckman	TuThu 11:30
7	Duanmu	TuThu 8:30	16	Wen	TuThu 1:00
8	Oloo	TuThu 10:00	17	Wen	TuThu 2:30

- No papers or notes may be used.
- Please don't just give an answer. Clearly explain how you get it, providing appropriate mathematical details. **An answer of 'convergent' or 'divergent' with no supporting work will be awarded zero points**
- This is a 2 hour exam.

Question	Grade
MC Total	
6	
7	
8	
9	
Total (out of 100)	

**Multiple Choice Section:** Choose the one option that best answers the question. There is no partial credit for questions 1-5.

1. [5 points] Consider the following:

$$a_n = \frac{n^3}{3n^3 + 5}.$$

- I.  $a_n$  converges
- II.  $a_n$  diverges
- III.  $\sum_{n=1}^{\infty} a_n$  converges
- IV.  $\sum_{n=1}^{\infty} a_n$  diverges

Which of the following is true?

- (a) I & III
- (b) I & IV
- (c) II & III
- (d) II & IV

2. [5 points] Which of the following are true for the Test for Divergence:

- I. If  $\lim_{n \rightarrow \infty} a_n$  does not exist, then  $\sum_{n=0}^{\infty} a_n$  diverges.
- II. If  $\lim_{n \rightarrow \infty} a_n$  exists but is not 0, then  $\sum_{n=0}^{\infty} a_n$  diverges.
- III. If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\sum_{n=0}^{\infty} a_n$  converges.

- (a) I & II
- (b) I & III
- (c) I, II & III
- (d) II & III

3. [5 points] Consider two positive series  $\sum a_n$  and  $\sum b_n$  with  $a_n \leq b_n$ . Which of the following is definitely true?

- (a) If  $\sum a_n$  converges, then  $\sum b_n$  converges.
- (b) If  $\sum b_n$  diverges, then  $\sum a_n$  diverges.
- (c) If  $\sum b_n$  converges, then  $\sum a_n$  converges.
- (d) If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ , then both  $\sum a_n$  and  $\sum b_n$  converge or both diverge.

4. [5 points] Determine if the following series converges, and if so, the value it converges to.

$$\sum_{n=1}^{\infty} (-1)^n \frac{7 \cdot 4^n}{3^n}.$$

- (a) -21
  - (b) -4
  - (c) 3
  - (d) divergent
5. [5 points] Which of the following yields an *inconclusive* result for the Root Test?

- (a)  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = 1$
- (b)  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \infty$
- (c)  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L < 1$
- (d)  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L > 1$

Please fill in your letter answer for questions 1-5 below:

(1) ----- (2) ----- (3) ----- (4) ----- (5) -----

**Free Response Portion:** Show all work for each of the following questions. Partial credit may be awarded for questions 6-9. You will receive no credit for an answer of “convergent” or “divergent” without supporting work.

6.

(a) [5 points] Evaluate the following integral.

$$\int \frac{y}{y^2 - 2y - 3} dy$$

(b) [10 points] Does the following integral converge or diverge? If it converges, what does it converge to?

$$\int_2^3 \frac{1}{\sqrt{3-x}} dx$$

7.

(a) [10 points] Find the values of  $x$  for which the series is convergent.

$$\sum_{n=0}^{\infty} (3)^{n+1} (x+4)^n$$

(b) [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=1}^{\infty} \frac{\cos^2(n)}{5^n}$$

8.

- (a) [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=1}^{\infty} \frac{4n}{6n^2 + 2n + 8}$$

- (b) [10 points] Does the series converge or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=3}^{\infty} \frac{\ln(n^2)}{n}$$

9.

- (a) [10 points] Does the series absolutely converge, conditionally converge, or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=1}^{\infty} \frac{n^3(n+1)!}{e^n n!}$$

- (b) [10 points] Does the series absolutely converge, conditionally converge, or diverge? State which test you used and clearly show that the series meets the conditions to use this test.

$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{5}{3\sqrt{n} - 3}$$



*This page is intentionally left blank for additional work.*