MATH 455 PROBLEM SET HINTS

PROBLEM SET 7

These are (usually) not complete solutions for the problems, but are intended to give you the basic ideas needed for a solution. If the basics of a problem are covered in class, either through working it out or doing a similar example, then we omit it here. Complete solutions typically involve more writing than is given here.

§2.5.

(1) $50 - 30 - 26 - 12 + 9 + 23 + 3 - 1 = 16$

(3) The properties are “has meat”, “has vegetable”, “is fried”. You’re given the sizes of all the subsets in the theorem except for one (“has vegetable”).

(6) Number the suits 1, 2, 3, 4. Let $a_i, i = 1, \ldots, 4$ be the property that a given hand contains no cards of suit $i$. We have $N_i = \binom{39}{5}, N_{ij} = \binom{26}{5}$, and $N_{ijk} = \binom{13}{5}$. Thus the number with at least one from each suit is $\binom{52}{5} - 4 \binom{39}{5} + 6 \binom{26}{5} - 4 \binom{13}{5} = 685464$.

(10) For these, do inclusion/exclusion over the set of edges. In other words, let $E$ be the set of edges of the graph. For any subset $A \subseteq E$, let $c(A)$ be the number of connected components of the subgraph with edge set $A$. For instance, $c(\emptyset)$ is the number of vertices. The number of colorings with $t$ colors that are the same along the subgraph $A$ is then $t^{c(A)}$. So the chromatic polynomial is $C_G(t) = \sum_{A \subseteq E} (-1)^{|E-A|} t^{c(A)}$.

(a) $t^4 - 4t^3 + 5t^2 - 2t$

(b) $t^5 - 6t^4 + 15t^3 - 17t^2 + 7t$

(11) Not really. According to the last bit of 2.5, the probability for 12 is $0.36787944132128159905937683715461493239$, and that for 120 is $0.36787944117144232159552377016146086745$. The difference is about $10^{-10}$.