Errata for Papantonopoulou text

• Problem 2.5.10 asks the student to show that the automorphism group of $\mathbb{Z}_p$ is isomorphic to $\mathbb{Z}_{p-1}$, where $p$ is prime. The book has already shown that the automorphism group is the group of units of $\mathbb{Z}_p$, so the students know that the order is $p - 1$. But showing that it’s cyclic is, as far as I know, beyond anything they can do at this point.

• Problem 4.4.16 says “Let $G$ be a group acting on itself by conjugation. Show that if $a$ and $b$ are conjugates in $G$, then the centralizers $C(a)$ and $C(b)$ are equal if and only if these centralizers are normal subgroups of $G$.” One direction is easy, of course, but the other direction doesn’t seem to be true even if you assume $a$ and $b$ are distinct—e.g., $a = (123)$ and $b = (132)$ in $S_6$.

• Definitions 7.2.22 and 7.2.23 (of prime and maximal ideals) require the ideals to be nontrivial. But then Theorem 7.2.27, which says that (in a commutative ring with 1), an ideal is prime (maximal) if and only if the quotient mod that ideal is a domain (field) isn’t right.

• Problem 8.3.23 The polynomial doesn’t have to be of degree $n$ even if it’s not the 0 polynomial, just of degree at most $n$. And if all the $b_i$ are 0, you get the 0 polynomial, which doesn’t have degree.

• Definition 10.1.2 (of vector space) requires $a \cdot (v + w) = (a \cdot v) + (a \cdot w)$ but not $(a + b) \cdot v = (a \cdot v) + (b \cdot v)$ I think both distributive laws are needed.

• In problem 10.2.1, I’m not sure how the students are expected to compute the degree of $\sqrt{2} + \sqrt{3}$ over $\mathbb{Q}(\sqrt{5})$. This seems like it’s much more substantial than the other parts of the problem, at least if done carefully.

• The subfield lattice in the solutions manual for problem 10.3.14 isn’t right. It ought to have a $\mathbb{Q}(\sqrt{3})$, for example.

• In the definition of the Frobenius map just before Prop. 10.4.8, a $\varphi$ gets switched to an $f$.

• In 11.1, there’s a missing statement of a proposition (that sum/difference of constructible numbers is constructible?).

• On page 430 in the proof of Galois’s theorem, the quotient group $G(L/E)/G(l/F)$ has the numerator and denominator reversed.