MATH 455 TUTTE POLYNOMIAL PROBLEMS

(1) Compute the Tutte polynomials of the following graphs:
   (a) The n-cycle C_n.
   (b) $K_4 - e$, where e is an edge
   (c) The graph in Figure 1
   (d) $K_{3,3}$

(2) Let G be a graph, and let G' be the graph obtained by subdividing one edge of G into a path of length 2.
   (a) Find a relationship between the Tutte polynomials of G and G'.
   (b) Use your relationship to compute the Tutte polynomial of C_6 by starting with C_3, which has Tutte polynomial $x^2 + x + y$.

(3) Let $T_G(x, y)$ be the Tutte polynomials of a connected graph G. Then one can show that $T_G(1, 1)$ is the number of spanning trees of G, and $T_G(1, 2)$ is the number of connected spanning subgraphs (a subgraph $H \subset G$ is spanning if H contains all the vertices of G; H doesn't have to be a tree). Check these results for the complete graph K_4.

(4) Tutte's original definition of his polynomial was as follows. Let G be a graph with vertices V and edge set E. For any subset $A \subset E$, let $k(A)$ be the number of connected components of the graph with vertices V and edges A. For any set S, let $|S|$ be the size of S. Then Tutte defined

$$T_G(x, y) = \sum_{A \subseteq E} (x-1)^{k(A)-k(E)} (y-1)^{k(A)+|A|-|V|}$$

The sum is taken over all subsets of E, including E itself and the empty set.
   (a) Apply Tutte's original definition to the cycle C_3, and show that you get the correct answer $x^2 + x + y$.
   (b) (Extra credit; not part of the official assignment) Show that Tutte's definition agrees with the definition given in class.

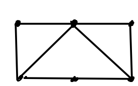

FIGURE 1.