MATH 455 EXAM II

This exam is worth 100 points, with each problem worth 20 points. There are problems on *both sides* of the page. Please complete Problem 1 and then *any four* of the remaining problems. Unless indicated, you must justify your answer to receive credit for a solution; correct answers alone are not necessarily sufficient for credit.

When submitting your exam, please indicate which problems (including Problem 1) you want graded by writing them in the upper right corner on the cover of your exam booklet. You must select exactly five problems; any unselected problems will not be graded, and if you select more than five only the first five (in numerical order) will be graded.

(1) Please classify the following statements as *True* or *False*. Write out the word completely; do not simply write T or F. There is no partial credit for this problem, and it is not necessary to show your work for this problem.
- (a) (4 pts) The coefficients of the chromatic polynomial of any graph are positive integers.
- (b) (4 pts) The binomial coefficients satisfy the identity $\sum_{k=0}^{n} \binom{n}{k} = 2^n$ for all integral $n \geq 0$.
- (c) (4 pts) The coefficient of x^4y^6 in $(x+y+z)^{10}$ is 210.
- (d) (4 pts) According to Kuratowski's theorem, every graph can be colored with at most 4 colors.
- (e) (4 pts) The reliability polynomial $R_G(p)$ of any graph vanishes at $p = 1$.

(2) (20 pts) Is the *Heawood graph* in Figure **??** planar? If yes, draw a planar embedding. If no, explain why not.

(3) Suppose G' is obtained from G by attaching a 3-cycle at a single vertex. (Figure **??**)
- (a) (10 pts) Show that the Tutte polynomial $T_{G'}(x,y)$ equals $(x^2+x+y)T_G(x,y)$, where $T_G(x,y)$ is the Tutte polynomial of G.
- (b) (10 pts) Compute the Tutte polynomial of the graph H shown in Figure **??**

(4) (20 pts) One of the following is the chromatic polynomial of a graph, and the others are not. Find the ones that aren't and explain why they can't be chromatic polynomials.
- (a) $k^2 - 2k + 1$
- (b) $k^3 - 4k^2 + 2k$
- (c) $k^3 - 3k^2 + k$
- (d) $k^4 - 5k^3 + 8k^2 - 4k$
- (e) $k^4 - 3k^3 + k^2 + k$

(5) Verify the following binomial coefficient identities. You may use any method of proof you like.
- (a) (6 pts) $k\binom{n}{k} = n\binom{n-1}{k-1}$
- (b) (7 pts) $\sum_{k=0, k \text{ even}}^{n} \binom{n}{k} = \sum_{k=0, k \text{ odd}}^{n} \binom{n}{k}$
- (c) (8 pts) $\sum_{k=1}^{n} k\binom{n}{k} = n \cdot 2^{n-1}$

(6) Consider the word *ROCOCO*.
- (a) Count all possible words of length 6 that can be made from this word.
- (b) Count all possible words of length 3 that can be made from this word.

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(c) Count all possible words of length 6 that can be made, if we reqire that all three Os should be adjacent.

(7) (20 pts) A ballot lists ten candidates for city council, eight candidates for the school board, and five bond issues. The ballot instructs voters to choose up to four people running for city council, rank up to three candidates for the school board, and approve or reject each bond issue. How many different ballots may be cast, if partially completed (or empty) ballots are allowed?

(8) (20 pts) A political science quiz has two parts. In the first, you must present your opinion of the four most influential secretaries-general in the history of the United Nations in a ranked list. In the second, you must name ten members of the United Nations security council in any order, including at least two permanent members of the council. If there have been eight secretaries-general in U.N. history, and there are fifteen members of the U.N. security council, including the five permanent members, how many ways can you answer the quiz, assuming you answer both parts completely?

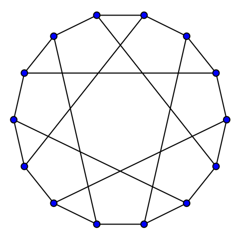

FIGURE 1. Planar or not?

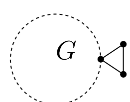

FIGURE 2. Gluing a 3-cycle onto a graph G

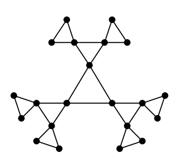

FIGURE 3.