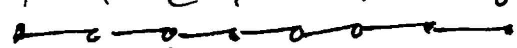
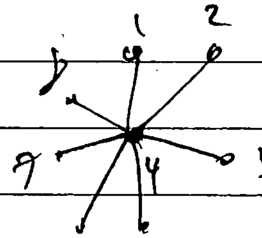
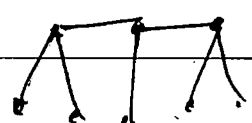
Exam I Answers

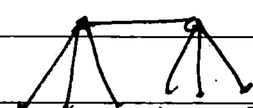
(1)
(a) F • • must be connected.
(b) F jumble of terms.
(c) F must be connected • •
(d) T thm from class
(e) F  K_n $n \geq 3$, C_n $n \geq 3$,
...

(2) all have length 6 $\Rightarrow$ all trees will have 8 verts.

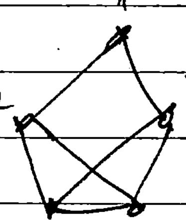
(a) 

(c) 

(b) 

(d) 

do algorithm from class.

(3) 

want # of spanning trees.
use matrix-tree thm.
D − A

$$\begin{pmatrix} 2 & & & & \\ & 3 & & & \\ & & 3 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

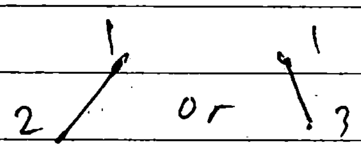
$$= \begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 \\ -1 & 0 & 3 & -1 & -1 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{pmatrix}$$

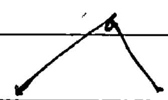
Find |det| of 4x4 submat.

$$\det \begin{pmatrix} -1 & 3 & 0 & -1 \\ -1 & 0 & 3 & -1 \\ 0 & -1 & -1 & 3 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -3 & 0 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & -1 & -1 & 3 \\ 0 & -1 & -1 & -1 \end{pmatrix}$$

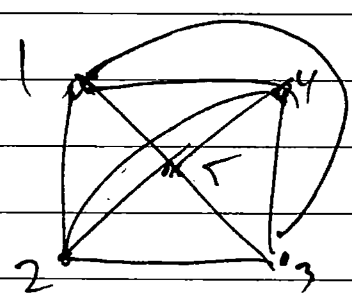
... 24

 ⇒ no. 8

 ⇒ 16

h.ll  = 8

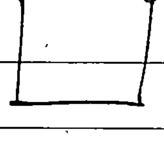
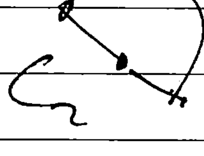
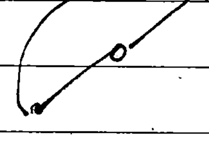
(4)



every vert has even degree ⇒ Eulerian.

1st do disjoint cycles

14321 1351. 2542

C_1  C_2  C_3 

C_3 plug in

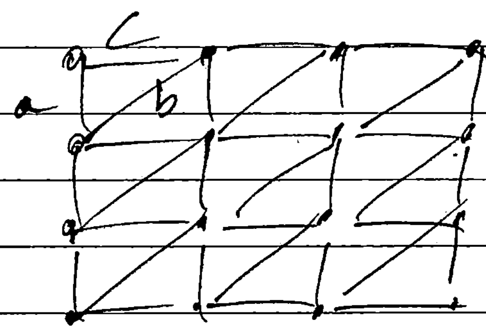
C_1, C_2

14325421351

C_2 then C_1, C_3 13514325421

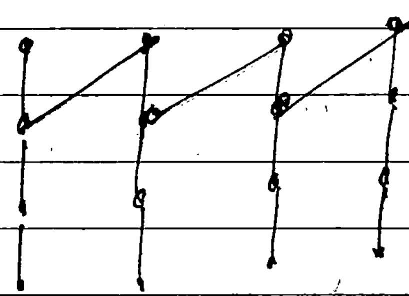
2 diff. ones.

5



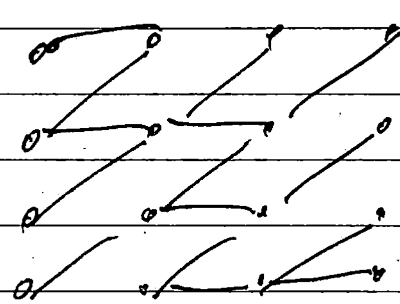
Find MST.

a $0 < a < b < c$.



not unique

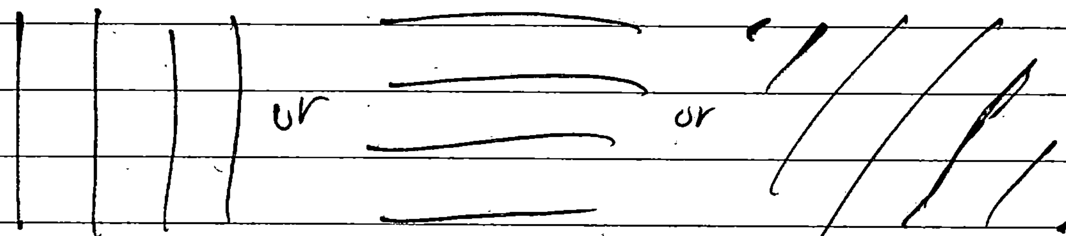
b $0 < b < c < a$.



not unique

(c) NO. enough to check all 3 a, b, c distinct.

We always take all edges of min wt.



always have choice after this

(6) (a) K_4 $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = A$

(b) $A^2 \begin{pmatrix} 0 & 1 & 1 & 1 \\ & \cdots & & \end{pmatrix} \begin{pmatrix} 0 & 1 & & \\ 1 & 0 & \cdots & \\ 1 & 1 & & \\ 1 & 1 & & \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{pmatrix}$

$A^3 \begin{pmatrix} 0 & 1 & 1 & 1 \\ & \cdots & & \end{pmatrix} \begin{pmatrix} 3 & 2 & \\ 2 & 3 & \\ 2 & 2 & \cdots \\ 2 & 2 & \end{pmatrix} = \begin{pmatrix} 6 & 7 & \cdots \\ 7 & & \\ \vdots & & \end{pmatrix}$

α: 0, 3, 6

β: 1, 2, 7

use inducho

(c) $\begin{pmatrix} a+1 & a & a & a \\ & \dots & & \end{pmatrix}\begin{pmatrix} 0 & 1 & \\ 1 & 0 & \dots \\ 1 & 1 & \\ 1 & 1 & \end{pmatrix} = \begin{pmatrix} 3a & 3a+1 & \dots \\ & & \end{pmatrix}$

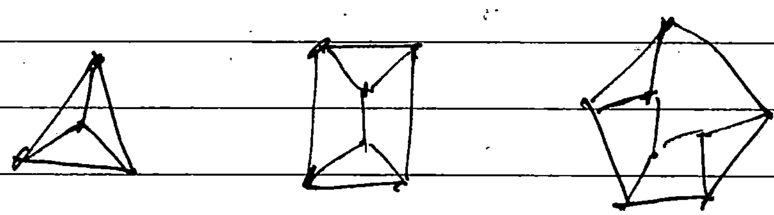
or $\begin{pmatrix} a & a+1 & a+1 & a+1 \\ & & & \end{pmatrix}\begin{pmatrix} 0 & 1 & \\ 1 & 0 & \dots \\ 1 & 1 & \\ 1 & 1 & \end{pmatrix} = \begin{pmatrix} 3a+3 & 3a+2 \\ & \end{pmatrix}$

7 (a) $\sum_v \deg(v) = 2E$

$\deg(v) = 3$, odd number of verts,

$\Rightarrow$ can't $2E$ cause will be odd.

(b)



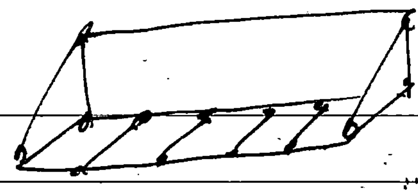
K_4 K_d.

$\downarrow$

(c) use formula from (a) get.

$2E = d^2 \Rightarrow E = \frac{d^2}{2} ? \frac{d(d-1)}{2}$

(d)



etc.