MATH 455 PROBLEM SET HINTS

PROBLEM SET 7

These are (usually) not complete solutions for the problems, but are intended to give you the basic ideas needed for a solution. If the basics of a problem are covered in class, either through working it out or doing a similar example, then we omit it here. Complete solutions typically involve more writing than is given here.

§2.5.

(1) $50 - 30 - 26 - 12 + 9 + 23 + 3 - 1 = 16$

(3) The properties are "has meat", "has vegetable", "is fried". You're given the sizes of all the subsets in the theorem except for one ("has vegetable").

(6) Number the suits $1, 2, 3, 4$. Let $a_i, i = 1, \ldots, 4$ be the property that a given hand contains no cards of suit i. We have $N_i = \binom{39}{5}$,$N_{ij} = \binom{26}{5}$, and $N_{ijk} = \binom{13}{5}$. Thus the number with at least one from each suit is $\binom{52}{5} - 4\binom{39}{5} + 6\binom{26}{5} - 4\binom{13}{5} = 685464$.

(10) For these, do inclusion/exclusion over the set of edges. In other words, let E be the set of edges of the graph. For any subset $A \subseteq E$, let $c(A)$ be the number of connected components of the subgraph with edge set A. For instance, $c(\emptyset)$ is the number of vertices. The number of colorings with t colors that are the same along the subgraph A is then $t^{c(A)}$. So the chromatic polynomial is

$$C_G(t) = \sum_{A \subseteq E} (-1)^{|E-A|} t^{c(A)}.$$

(a) $t^4 - 4t^3 + 5t^2 - 2t$

(b) $t^5 - 6t^4 + 15t^3 - 17t^2 + 7t$

(11) Not really. According to the last bit of 2.5, the probability for 12 is 0.36787944132128159905937683715461493239, and that for 120 is 0.36787944117144232159552377016146086745. The difference is about 10^{-10}.