MATH 455 PROBLEM SET HINTS

PROBLEM SET 4

These are (usually) not complete solutions for the problems, but are intended to give you the basic ideas needed for a solution. If the basics of a problem are covered in class, either through working it out or doing a similar example, then we omit it here. Complete solutions typically involve more writing than is given here.

§1.4.1.

(3) Only two land masses meet an odd number of bridges, so ...
(4) Make a graph where the intersections are the vertices and the roads are the edges. Two vertices then have odd degree.

§1.4.2.

(1) (a) An even cycle.
 (b) Paste an odd and an even cycle together along a common vertex.
 (c) Paste two odd cycles together along a common vertex.
 (d) An odd cycle.
(2) I'll leave the final answer to you.
(6) $L(G)$ is connected if G is connected. $L(G)$ has a vertex for every edge and an edge for every pair of edges that meet in a vertex of G. If G is regular of degree r, then every vertex of $L(G)$ will meet $2r - 2$ edges in $L(G)$. Thus every vertex has even degree in $L(G)$.
(7) I will leave (a) to you. For (b), we need every vertex to be even degree. This will happen if and only if n_1 and n_2 are even, since every vertex in K_{n_1,n_2} has degree n_1 or n_2.

§1.5.1.

(1) See Figures 1 and 2.
(2) I did this by enlarging the polygon bounding a given region and "folding" the rest of the graph into the interior. Figure 3 shows the result for R_3. (I added the vertex labels for my own benefit.)
(7) See Figure 4 (sorry the labels are numbers, not letters).

§1.5.2.

(1) Order 24 and regularity 3 implies that there are 36 edges. Since $v - e + f = 2$, we have 14 regions.
(3) Any counterexample is good. For instance let G be two different vertices and no edges. Then $v - e + f = 2 - 0 + 1 = 3 \neq 2$.

(10) One is given by the vertices and edges of the octahedron. But there are infinitely many 4-regular planar graphs. (If you want extra credit try to convince me of it with an original argument.)

§1.5.3.

(1) $K_{2,2,2}$

(2) Lots of possibilities. How about a soccer ball? Look on Wikipedia, say under *Archimedian solids*. In general you can take a bunch of random points on a sphere and then form their convex hull (the smallest convex set containing them) to make as many examples as you want.

§1.5.4.

(2) This is challenging, but it can be done. Label the vertices of the outer pentagon as a, b, c, d, e starting from the 12 o'clock position and going clockwise, and label the vertices of the inner pentagon as A, B, C, D, E in the same fashion. Erase the edges bB and AC. The vertices A, B, C, b can now be erased to give $K_{3,3}$.

(4) Assume that $n > 1$, since we know which complete graphs are planar. Then as soon as any two of the r_i are bigger than 2, the graph will contain $K_{3,3}$ as a subgraph and can't be planar. $K_{2,2}$ and $K_{2,2,2}$ are planar. What about $K_{2,2,2,2}$?

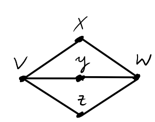

FIGURE 1.

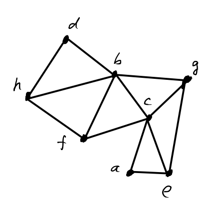

FIGURE 2.

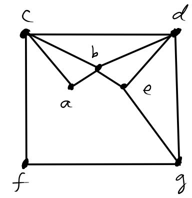

FIGURE 3.

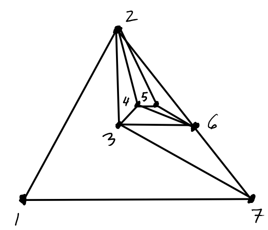

FIGURE 4.