MATH 455 PROBLEM SET HINTS

PROBLEM SET 5

These are (usually) not complete solutions for the problems, but are intended to give you the basic ideas needed for a solution. If the basics of a problem are covered in class, either through working it out or doing a similar example, then we omit it here. Complete solutions typically involve more writing than is given here.

§1.6.1.

(1) (a) 1 for K_1, 2 otherwise.
 (b) 2.
 (c) t (you need one color for each r_i).
 (d) 3.
 (e) 3.
 (f) tetra: 4. cube: 2. octa: 3. dodeca: 3. icosa: 4. These are found by giving explicit colorings.

(3) The chromatic number of the associated graph is 3. The groups are Moe, Mike, Jeff; Larry, John; and Curly has to be by himself.

(4) Suppose the chromatic number is k and the two verts are v and w. If there is a k-coloring where v and w get different colors, then the chromatic number doesn't increase when the edge between v and w is added. Otherwise, if every k-coloring gives v and w the same colors, then we need a new color when we join v and w. So the number goes up by at most 1.

§1.6.2.

(2) The largest degree of a vertex is 7 so $\omega(G) \leq 8$. It can't be 8 since it's not K_8. The other degrees aren't big enough to make it include K_7. It can't include K_6 for the same reason. So the biggest we can hope for is $\omega = 5$. And you can see K_5 in there, thus $\omega = 5$. To see that $\chi = 6$, notice that we need three colors on the right column and at least three on the left column (because those are odd cycles). You can check that 6 colors are enough.

(4) Suppose P is a detour path. If $\chi > \tau$, then there must be a vertex v that needs a different color from all the vertices in P. But this means it must be connected to all the vertices in P. So we can lengthen P by adding a new edge on the end, which contradicts P being a detour path.

§1.6.3.

(1) We know it's at most 4. We just need to make sure that it's not less than 4. Take a look at Kentucky (for instance). It borders 7 states, and they form an odd cycle around it. So we need 4 colors (because odd cycles need at 3 colors, and we need another for Kentucky).

(2) This time the chromatic number is 4 again. Look at Paraguay. It borders Brazil, Bolivia, and Argentina, and they border each other in pairs. So you need 4 colors. (I hope people printed out a map and tried to color it . . . you'd find this configuration that way.)

§1.6.4.

(1) (a) $t^4 - 3t^3 + 3t^2 - t$
    (b) $t^6 - 5t^5 + 10t^4 - 10t^3 + 5t^2 - t$
    (c) $t^4 - 4t^3 + 6t^2 - 3t$
    (d) $t^5 - 5t^4 + 10t^3 - 10t^2 + 4t$
    (e) $t^4 - 5t^3 + 8t^2 - 4t$
    (f) $t^5 - 9t^4 + 29t^3 - 39t^2 + 18t$

(2) At $k = 2$ we get -4. The values of chromatic polynomials at positive integers are always ≥ 0.