MATH 455 PROBLEM SET HINTS

PROBLEM SET IV

These are (usually) not complete solutions for the problems, but are intended to give you the basic ideas needed for a solution. If the basics of a problem are covered in class, either through working it out or doing a similar example, then we omit it here. Complete solutions typically involve more writing than is given here.

§1.5.1.

(1) See Figures 1 and 2.
(2) I did this by enlarging the polygon bounding a given region and "folding" the rest of the graph into the interior. Figure 3 shows the result for R_3. (I added the vertex labels for my own benefit; as was pointed out in class this is helpful to do.)
(7) See Figure 4 (sorry the labels are numbers, not letters).

§1.5.2.

(1) Order 24 and regularity 3 implies that there are $36 = 3 \cdot 24/2$ edges. Since $v - e + f = 2$, we have 14 regions.
(3) I tried to explain this in class, but it seemed like no one understood me at all. (Not the first time that's happened). Here's yet another attempt. The surface of the earth is a sphere. So if you draw a planar graph on a small, flat surface on the earth (like on a sidewalk with sidewalk chalk) you have embedded the planar graph in the sphere. Conversely suppose we have a graph drawn on the surface of the earth. Pick a point that is in one of the regions and shrink the graph to a smaller size by flowing away from that point. Eventually the graph will be small enough to be drawn on a small flat surface, so it can be embedded in a plane. What I'm describing is essentially the same thing as "stereographic projection"; you can read about this on Wikipedia.
(10) One is given by the vertices and edges of the octahedron. But there are infinitely many 4-regular planar graphs (examples of infinite families are easily found online).

§1.5.3.

(1) $K_{2,2,2}$
(2) Lots of possibilities. How about a soccer ball? Look on Wikipedia, say under *Archimedian solids*.

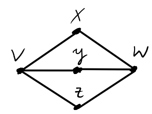

FIGURE 1.

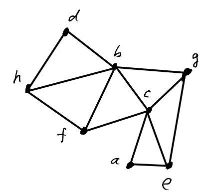

FIGURE 2.

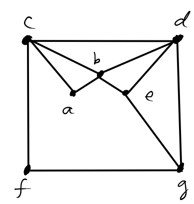

FIGURE 3.

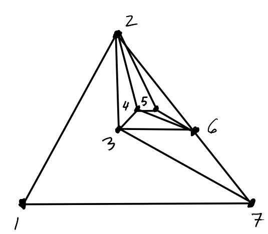

FIGURE 4.

§1.5.4.

(2) Done in class. (Try to recover a $K_{3,3}$.)

(4) Assume that $n > 1$, since we know which complete graphs are planar. Then as soon as any two of the r_i are bigger than 2, the graph will contain $K_{3,3}$ as a subgraph and can't be planar. $K_{2,2}$ and $K_{2,2,2}$ are planar. What about $K_{2,2,2,2}$?