MATH 455 PROBLEM SET HINTS

PROBLEM SET III

These are (usually) not complete solutions for the problems, but are intended to give you the basic ideas needed for a solution. If the basics of a problem are covered in class, either through working it out or doing a similar example, then we omit it here. Complete solutions typically involve more writing than is given here.

§1.3.4.

(2) (a) 221314415. (b) 5843339.
(3) See Figure 1.

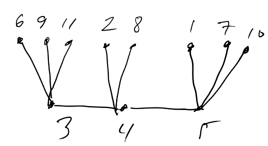

FIGURE 1.

(4) $K_{1,n}$, i.e. stars.
(5) Path graphs.

§1.4.1.

(3) Only two land masses meet an odd number of bridges, so when we draw the graph underlying the situation, we find two vertices of odd degree. This means there is an Eulerian trail in the graph.
(4) Make a graph where the intersections are the vertices and the roads are the edges. Two vertices then have odd degree (corresponding to the middle top and the middle bottom intersection). Thus an Eulerian trail exists. Finding it can be done by trial and error, or by using the techniques in the next section.

§1.4.2.

(1) (a) An even cycle.
 (b) Paste an odd and an even cycle together along a common vertex.
 (c) Paste two odd cycles together along a common vertex.
 (d) An odd cycle.
(2) A decomposition into disjoint cycles is given by $bcgfb$, $ghlkg$, $jkonj$, $efjie$, $abea$, $cdhc$, $lpol$, $inmi$.

(6) $L(G)$ is connected if G is connected. $L(G)$ has a vertex for every edge and an edge for every pair of edges that meet in a vertex of G. If G is regular of degree r, then every vertex of $L(G)$ will meet $2r-2$ edges in $L(G)$. Thus every vertex has even degree in $L(G)$.

(7) For (b), we need every vertex to be even degree. This will happen if and only if n_1 and n_2 are even, since every vertex in K_{n_1,n_2} has degree n_1 or n_2. For (a), we need two vertices of odd degree, so one of n_1 or n_2 must be odd and the other must equal two.