(1) Is it possible for a Cayley graph to be a complete graph? Either explain why it is not possible, or give for any $n$ a pair of a group $G$ and a generating set $S$ such that the corresponding Cayley graph is $K_n$.

(2) Compute the Cayley graph for $G = \text{SL}_2(\mathbb{Z}/3\mathbb{Z})$, where $S$ consists of the matrices

\[
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix}
\]

(3) Compute the Cayley graph for $G = S_4$ (symmetric group on 4 letters), where $S = \{(1,2), (2,3), (3,4)\}$. (Hint: the Cayley graph for $S_3$ given in class will appear a few times as a subgraph.)

(4) Given two groups $G, H$, the product group $G \times H$ consists of all pairs $(g, h), g \in G, h \in H$ with multiplication given componentwise: $(g, h) \cdot (g', h') = (gg', hh')$. If $S_G$ generates $G$ and $S_H$ generates $H$, then the set of pairs $S_G \times S_H$ will generate $G \times H$.

(a) Draw the Cayley graph of $(\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/4\mathbb{Z})$, where the generating set is taken to be $(1, 1), (-1, -1)$.

(b) Explain what happens for $(\mathbb{Z}/m\mathbb{Z}) \times (\mathbb{Z}/n\mathbb{Z})$, where again the generating set is taken to be $(1, 1), (-1, -1)$.