Problem 1 Solution Sheet 22 Students

(i) $\int_1^e \frac{\ln x}{x}\,dx \underset{\uparrow}{=} \int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}$

$u = \ln x$
$du = \frac{1}{x}\,dx$

(ii) $\int \frac{e^{2x}}{e^x+1}\,dx = \int \frac{u^2}{u+1}\frac{du}{u} = \int \frac{u+1-1}{u+1}\,du =$

$u = e^x$
$du = e^x dx = u\,dx$

$= \int 1.du - \int \frac{du}{1+u} = u - \ln(u+1) + C.$

(iii) $\int \frac{\sqrt{x+1}}{x}\,dx = 2\int \frac{u^2}{u^2-1}\,du =$

$x+1 = u^2$
$dx = 2u\,du$

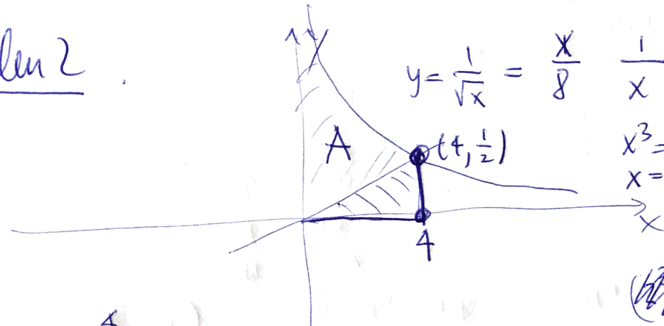
$= 2\int \frac{u^2-1+1}{u^2-1}\,du = 2\int du + 2\int \frac{du}{u^2-1} =$

$+$

$= 2u + \ln \frac{u-1}{u+1}$

$\frac{1}{u^2-1} = \frac{1}{2}\left(\frac{1}{u-1} - \frac{1}{u+1}\right)$

Problem 2




$y = \frac{1}{\sqrt{x}} = \frac{x}{8}$ $\quad \frac{1}{x} = \frac{x^2}{64}$

$x^3 = 64$

$x = 4$

$$A = \int_0^4 \frac{dx}{\sqrt{x}} - \underline{1} = 2\sqrt{x}\Big|_0^4 - 1 = 4 - 1 = 3$$

~~Problem 3~~ $\cancel{\gamma'(t) = (1-\cos t,\ 1+\sin t)}$ ~~velocity~~ ~~speed~~

$\cancel{\|\gamma'(t)\| = \sqrt{(1-\cos t)^2 + (1+\sin t)^2}}$

$\cancel{= \sqrt{1+\cos^2 t - 2\cos t + 1 + \sin^2 t + 2\sin t}}$

$\cancel{\sqrt{3 + 2(1-\cos t) + (1+2\sin t)}}$

# Problem 3

$$\gamma(t) = (t - \sin t, 1 - \cos t)$$

$$\gamma'(t) = (1 - \cos t, \sin t)$$

$$\|\gamma'(t)\| = \sqrt{1 + \cos^2 t - 2\cos t + \sin^2 t} = \sqrt{2(1 - \cos t)}$$

$$= \sqrt{2}\sqrt{2\sin^2\left(\tfrac{t}{2}\right)} = 2\sin t/2$$

$$L(\gamma) = 2\int_0^{2\pi} \sin t/2 \, dt = 2\left(-2\cos t/2\right)\Big|_0^{2\pi} =$$

$$= 2(2+2) = 8$$

# Problem 4

(i) a) vertical asymptote at $x=0$

~~b)~~ not improper

c) unbounded domain

d) unbounded domain & vertical asymptote at $x=0$

(ii) (iii) a) $\displaystyle\int_0^8 \frac{dx}{x^{1/3}} = \frac{3}{2} x^{2/3}\Big|_0^8 = \boxed{6 \text{ convergent}}$ a)

c) $\displaystyle\int_1^\infty \frac{2x^2+9x+4}{x^5}\,dx = \int_1^\infty \frac{2}{x^3}dx + \int_1^\infty \frac{9}{x^4}dx + \int_1^\infty \frac{4}{x^5}dx$

$= -2\,\frac{1}{2}x^{-2}\Big|_1^\infty - 9\,\frac{1}{3}x^{-3}\Big|_1^\infty - 4\,\frac{x^{-4}}{4}\Big|_1^\infty$

$= 1+3+1 = 5$ c) convergent

$\boxed{\text{d) diverged:}}$ $\displaystyle\int_0^\infty \frac{x+2}{x^3} = \int_0^1 \frac{x+2}{x^3} + \int_1^\infty \frac{x+2}{x^3}\,dx$

divergent. $\displaystyle\int_0^1 \frac{dx}{x^2} = -\frac{1}{x}\Big|_0^1 = \infty$ convergent ✓