MATH 331.02 Ordinary Differential Equation Professor Franz Pedit

Answer Key for **HW 8**

2. [10 pts]

(i) [5 pts] (1 pt each, 1/3 pts for each attempt, 1 pt for graph)

- (Four possible answers)

$$\begin{cases} y_{\gamma=10}(t) = \frac{9}{80}e^{-t} - \frac{1}{80}e^{-9t} \text{ (overdamped)} \\ y_{\gamma=6}(t) = \frac{1}{10}e^{-3t} + \frac{3}{10}e^{-3t}t \text{ (critically damped)} \\ y_{\gamma=\sqrt{20}}(t) = \frac{1}{10}e^{-\sqrt{5}t}\cos 2t + \frac{\sqrt{5}}{20}e^{-\sqrt{5}t}\sin 2t \text{ (underdamped)} \end{cases} \quad \{\text{in meters} - y(0) = 0.1\}$$

$$\begin{cases} y_{\gamma=10}(t) = \frac{45}{4}e^{-t} - \frac{5}{4}e^{-9t} \text{ (overdamped)} \\ y_{\gamma=6}(t) = 10e^{-3t} + 30e^{-3t}t \text{ (critically damped)} \\ y_{\gamma=\sqrt{20}}(t) = 10e^{-\sqrt{5}t}\cos 2t + 5\sqrt{5}e^{-\sqrt{5}t}\sin 2t \text{ (underdamped)} \end{cases} \quad \{\text{in centimeters} - y(0) = 10\}$$

$$\begin{cases} y_{\gamma=10}(t) = -\frac{9}{80}e^{-t} + \frac{1}{80}e^{-9t} \text{ (overdamped)} \\ y_{\gamma=6}(t) = -\frac{1}{10}e^{-3t} - \frac{3}{10}e^{-3t}t \text{ (critically damped)} \\ y_{\gamma=\sqrt{20}}(t) = -\frac{1}{10}e^{-\sqrt{5}t}\cos 2t - \frac{\sqrt{5}}{20}e^{-\sqrt{5}t}\sin 2t \text{ (underdamped)} \end{cases} \quad \{\text{in meters} - y(0) = -0.1\}$$

$$\begin{cases} y_{\gamma=10}(t) = -\frac{45}{4}e^{-t} + \frac{5}{4}e^{-9t} \text{ (overdamped)} \\ y_{\gamma=6}(t) = -10e^{-3t} - 30e^{-3t}t \text{ (critically damped)} \\ y_{\gamma=\sqrt{20}}(t) = -10e^{-\sqrt{5}t}\cos 2t - 5\sqrt{5}e^{-\sqrt{5}t}\sin 2t \text{ (underdamped)} \end{cases} \quad \{\text{in centimeters} - y(0) = -10\}$$

(ii) [3 pts] (1 pts each, 0.5 pts for attempt)

when $\gamma = 10$: $7.0255(s)$

when $\gamma = 6$: $3.0778(s)$

when $\gamma = \sqrt{20}$: $1.2010(s)$

(iii) (? Very good question lol)

(iv) [1 pts] $f = {}^{1}/_{\pi}\,(s^{-1})$

(v) [1 pts] $\gamma = 4$

3. [10 pts] (1 pt for attempt)

$$y(t) = e^{-2t}(\cos(t) + 4\sin(t)) = \begin{cases} \sqrt{17}e^{-2t}\cos(t - \arctan 4)\ (\approx 4.1231e^{-2t}\cos(t - 1.32582)) \\ \sqrt{17}e^{-2t}\sin\left(t + \arctan\frac{1}{4}\right)\ (\approx 4.1231e^{-2t}\sin(t + 0.24498)) \end{cases}$$

[6 pts] [3 pts]

$(1.325818\ \text{rad} \approx 75.96°, \quad 0.24498\ \text{rad} \approx 14.04°)$

6. [10 pts] (No attempt points)

(i) [2 pts] $y(t) = 3te^{-t}$

(ii) [2 pts]

$$y(1) = \frac{3}{e} \approx 1.10364$$

(iii) [3 pts] (1 pt for each graph; 1 pt for the explanation) (Explanation does not have to be lengthy – short brief explanations are still acceptable)

(iv) [1 pts] $t \approx 7.75169$ (s)

(v) [2 pts] "It will be returned eventually, when $t \to \infty$." *or* "It will not reach its original position but it will continuously approach toward it." ("No" or "Yes" are still acceptable)

In[1]:=
```
Clear[x, y, y1, y2, y3]
```
클리어

```
y1 = DSolve[{y''[x] + 10 y'[x] + 9 y[x] == 0, y[0] == 1/10, y'[0] == 0}, y[x], x]
```
미분 방정식

```
y2 = DSolve[{y''[x] + 6 y'[x] + 9 y[x] == 0, y[0] == 1/10, y'[0] == 0}, y[x], x]
```
미분 방정식

```
y3 = DSolve[{y''[x] + Sqrt[20] * y'[x] + 9 y[x] == 0, y[0] == 1/10, y'[0] == 0}, y[x], x]
```
미분 방정식 제곱근

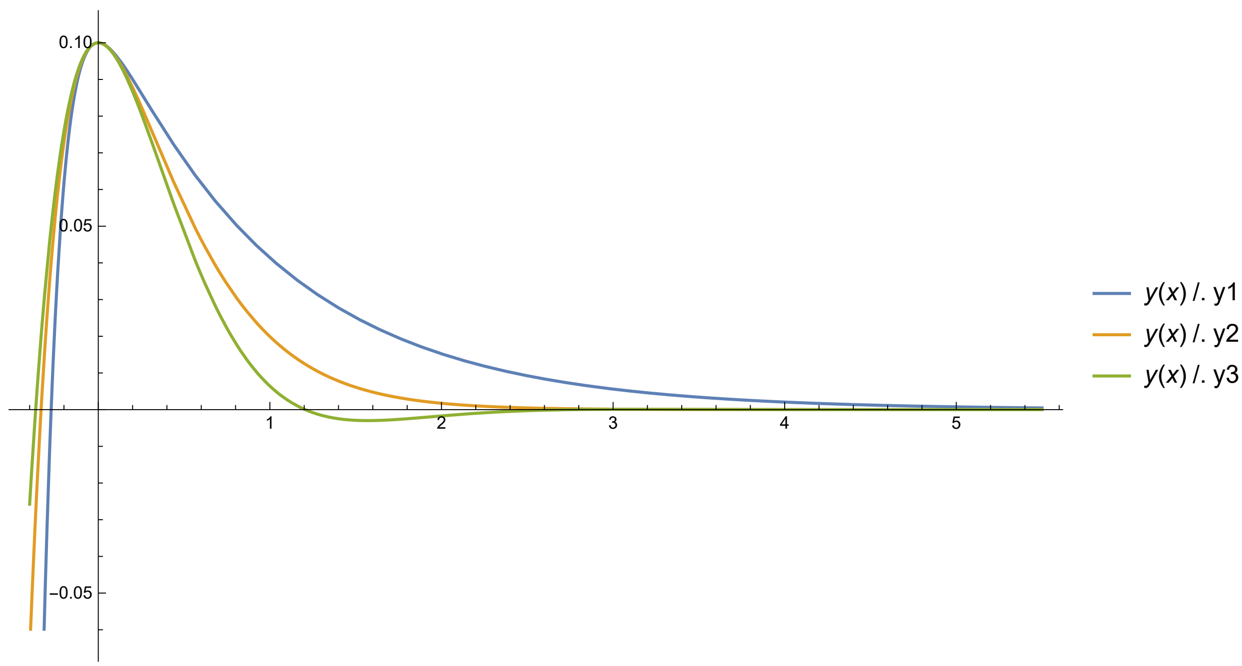
```
Plot[{y[x] /. y1, y[x] /. y2, y[x] /. y3}, {x, -0.4, 5.5}, PlotLegends → "Expressions"]
```
플롯 플롯 범례

Out[2]= $\left\{\left\{y[x] \to \frac{1}{80} e^{-9x} \left(-1 + 9 e^{8x}\right)\right\}\right\}$

Out[3]= $\left\{\left\{y[x] \to \frac{1}{10} e^{-3x} (1 + 3x)\right\}\right\}$

Out[4]= $\left\{\left\{y[x] \to \frac{1}{20} e^{-\sqrt{5}\,x} \left(2 \text{Cos}[2x] + \sqrt{5}\, \text{Sin}[2x]\right)\right\}\right\}$

Out[5]=



In[6]:=
```
Clear[x, y, y1, y2, y3]
```
클리어

```
y1 = DSolve[{y''[x] + 4 y'[x] + 5 y[x] == 0, y[0] == 1, y'[0] == 2}, y[x], x]
```
미분 방정식

Out[7]= $\left\{\left\{y[x] \to e^{-2x} (\text{Cos}[x] + 4 \text{Sin}[x])\right\}\right\}$

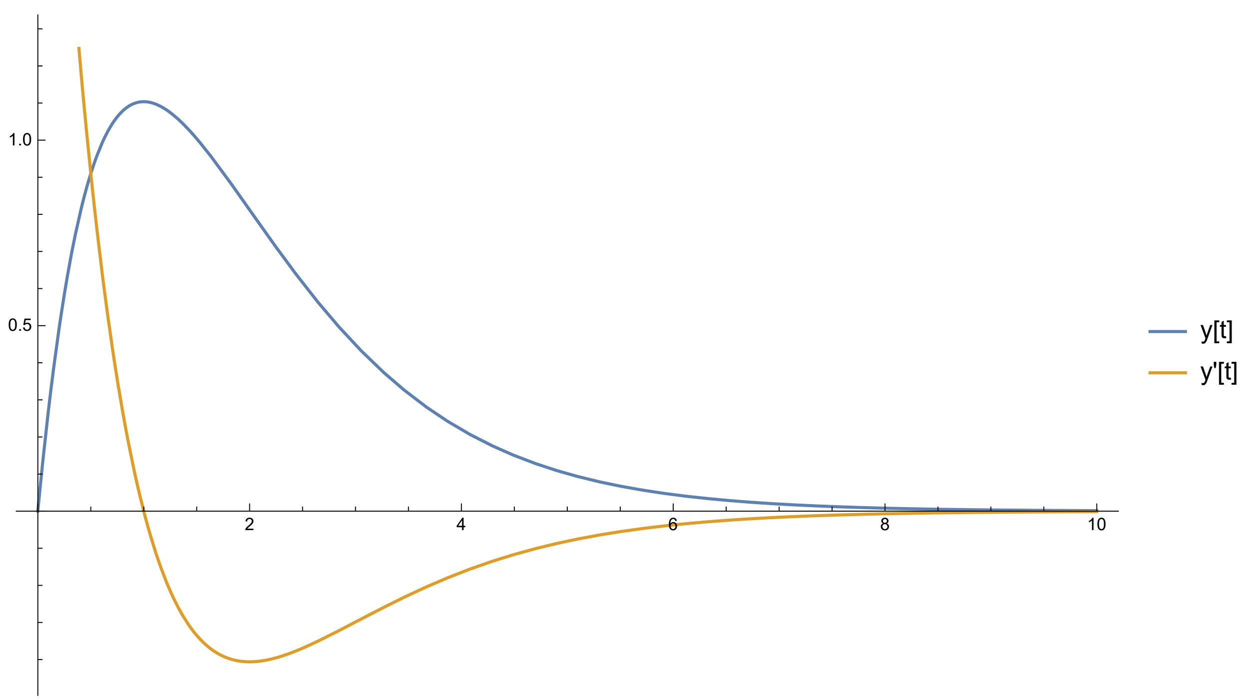
In[8]:=
```
Clear[x, y, y1, y2, y3]
클리어
y1 = DSolve[{10 y''[x] + 20 y'[x] + 10 y[x] == 0, y'[0] == 3, y[0] == 0}, y[x], x]
미분 방정식
FindMaximum[y[x] /. y1, {x, 0.1, 100}]
극대치를 추구
Clear[x, y]
클리어
y2 = D[y[x] /. y1, x]
미분 계수
Plot[{y[x] /. y1, y2}, {x, 0, 10}, PlotLegends → {"y[t]", "y'[t]"}]
플롯 플롯 범례
Clear[x, y]
클리어
FindRoot[y[x] == 0.01 /. y1, {x, 6}]
근 찾기
```

Out[9]= $\{\{y[x] \to 3\, e^{-x}\, x\}\}$

Out[10]= {1.10364, {x → 1.}}

Out[12]= $\{3\, e^{-x} - 3\, e^{-x}\, x\}$

Out[13]=



Out[15]= {x → 7.75169}