Homework 7, Advanced Calculus Due 4/7/17

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Consider the function $f(x) = \cos(x_1) + \sin(x_2)$.

- (i) Find all the critical points and critical values of f on \mathbb{R}^2 and draw a picture of the set of critical points in the (x_1, x_2) -plane.
- (ii) Characterize the critical points as local maxima, minima or saddle points.
- (iii) Find a global maximum and minimum of f inside the square (including the boundary) of side length π centered at the origin.

Problem 2. Which point(s) on the surface M given by the equations

$$x_1x_3 - x_2x_4 = 1 \qquad x_2x_3 + x_1x_4 = 0$$

are closest to the origin?

Problem 3. Consider the function $f(x_1, x_2) = x_1(x_1^2 - 4) + x_2(x_2^2 - 1)$. Calculate all critical points of f on \mathbb{R}^2 and characterize them as local maxima/minima or saddles. Does this function have a global maximum and/or minimum on \mathbb{R}^2 (justify your answer).

Problem 4. Consider a rectangular box (all faces are perpendicular and rectangles) of side lengths x, y, z. Assuming the volume is 1, which dimensions minimize (or perhaps maximize? Justify your answer) the total surface area of the box.

Problem 5. Consider the function $f(x) = x_1^2 + x_2^2 + 6x_1x_2$. Find the local maxima and minima of f along the circle of radius 2.