HOMEWORK 6, M 331.2
DUE 10/26/16

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Consider the linear 2nd order homogeneous ODE

$$y'' + 5y' + 6y = 0\,.$$

(i) Assuming there is a solution of the form $y(t) = e^{\lambda t}$ find the condition on λ such that this $y(t)$ solves the ODE, i.e. calculate the characteristic polynomial of the ODE and find its roots.
(ii) Find the solution $y(t)$ which satisfies the initial conditions $y(0) = y'(0) = 1$ (use linear superposition of the two solutions $y_1(t), y_2(t)$ you found in (i)).
(iii) Can you calculate a formula for the solution $y(t)$ which satisfies the initial conditions $y(0) = y_0$ and $y'(0) = v_0$? Once you have such a formula, plug in the previous initial data $y_0 = v_0 = 1$ and check whether you get the same answer.

Problem 2. Find the solution of the ODE

$$2y'' - 3y' + y = 0$$

with initial condition $y(0) = 1$ and $y'(0) = 0$.

Problem 3. Solve the initial value problem

$$y'' - 4y = 0\,, \qquad y(0) = 1\,, \quad y'(0) = v_0$$

and determine v_0 in such a way that this solution approaches zero as $t \to \infty$.

Problem 4. Find the solution of the ODE $y'' - 2y' - 3y = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.

Problem 5. Consider the homogeneous ODE $y'' + 6y' + 13y = 0$.

(i) Verify that this ODE has the two solutions $y_1(t) = e^{-3t}\sin 2t$ and $y_2(t) = e^{-3t}\cos 2t$.
(ii) Find the solution $y(t)$ with initial conditions $y(0) = 0$ and $y'(0) = 1$ (again, use linear superposition).
(iii) Calculate $\lim_{t\to\infty} y(t)$ for your solution in (ii).

Problem 6. This is similar to the previous problem but put in a mechanical context. Consider the mass-spring system pictured below, i.e., a mass m attached to a spring with spring constant k sliding on a surface with friction γ. The ODE for the position $y(t)$ of the mass, derived from Newtons equation of motion, is

$$my'' + \gamma y' + ky = 0$$

Assume $m = 1$, $\gamma = 2$, $k = 5$, time t is measured in seconds and y is measured in inches.

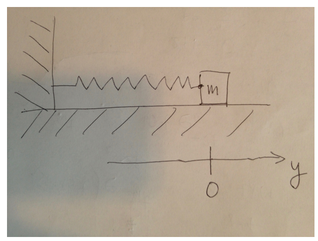
(i) Verify that $y_1(t) = e^{-t}\sin 2t$ and $y_2(t) = e^{-t}\cos 2t$ are solutions to the ODE.
(ii) Find the solution of the ODE with initial condition $y(0) = 0$ and initial velocity $y'(0) = 10$ (linear superposition tells us, that $y(t) = c_1y_1(t) + c_2y_2(t)$ is also a solution for any choice of c_1 and c_2). Interpret the initial conditions in words.

(iii) Describe the motion of the mass: what is the furthest distance the mass reaches from its resting position $y = 0$ and how long does it take to get there? What is the velocity of the mass at that moment?

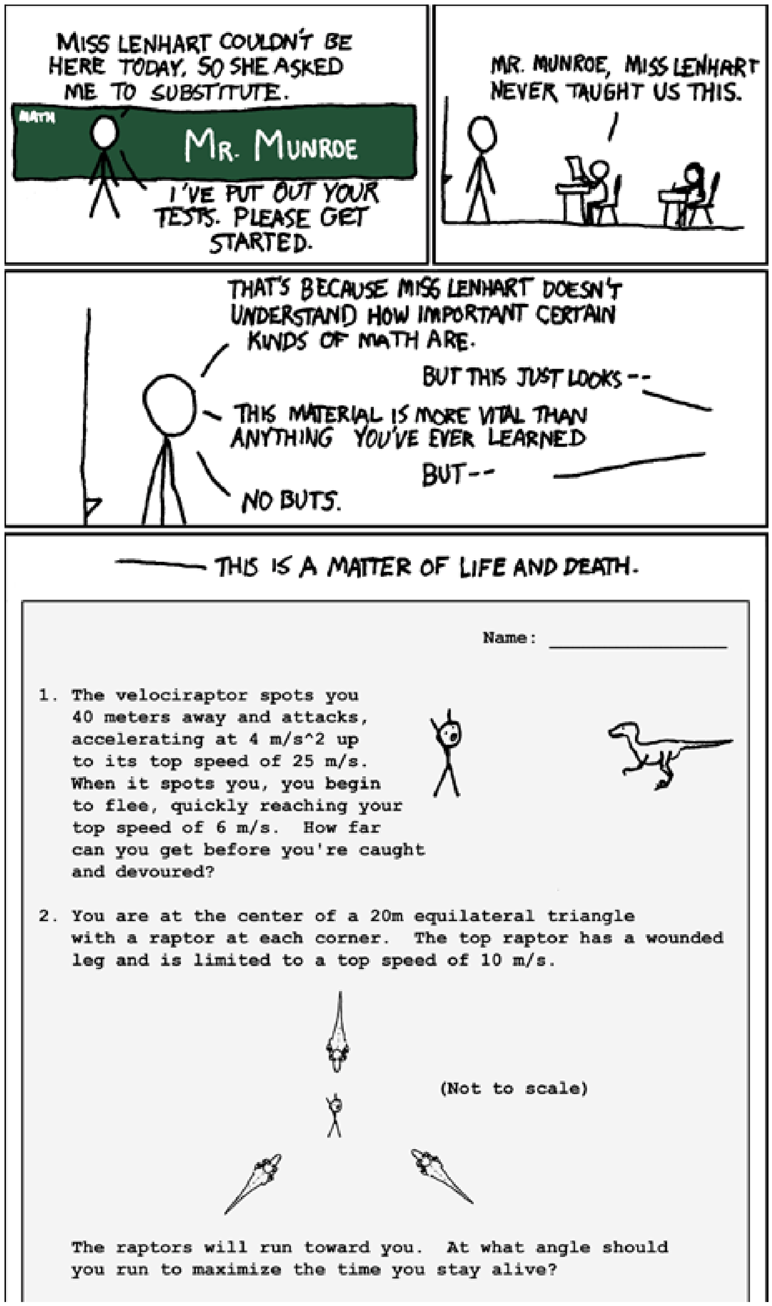
(iv) Is there a point in time when the acceleration is zero? What does this mean for the velocity?

(v) Does the mass ever reach its resting position again? After reaching its maximal distance from the resting position, how long will it take for the mass to come within 1/10 of an inch of the resting position?

(vi) Draw accurate (labeled, with units) graphs of the position $y(t)$ and velocity $y'(t)$ functions for $t \geq 0$ and picture the answers of the above question on those graphs.



Problem 7. Problem is explained on the next page....do the first one, and contemplate what you would do on the second question....


MISS LENHART COULDN'T BE HERE TODAY, SO SHE ASKED ME TO SUBSTITUTE.
MATH
MR. MUNROE
I'VE PUT OUT YOUR TESTS. PLEASE GET STARTED.
MR. MUNROE, MISS LENHART NEVER TAUGHT US THIS.
THAT'S BECAUSE MISS LENHART DOESN'T UNDERSTAND HOW IMPORTANT CERTAIN KINDS OF MATH ARE.
BUT THIS JUST LOOKS --
THIS MATERIAL IS MORE VITAL THAN ANYTHING YOU'VE EVER LEARNED
BUT--
NO BUTS.
THIS IS A MATTER OF LIFE AND DEATH.
Name: ________________
1. The velociraptor spots you 40 meters away and attacks, accelerating at 4 m/s^2 up to its top speed of 25 m/s. When it spots you, you begin to flee, quickly reaching your top speed of 6 m/s. How far can you get before you're caught and devoured?
2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.
(Not to scale)
The raptors will run toward you. At what angle should you run to maximize the time you stay alive?