# Homework 5, Honors Calculus II
## due 10/11/18

*Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top. All integrals have to be computed without using symbolic calculators. You may use a calculator only to verify a result, and for numerical calculations which you cannot do on paper or in your head.*

**Problem 1.** Calculate the surface area of a sphere of radius $R > 0$ by viewing the sphere as a surface of revolution.

**Problem 2.** Calculate the volume of a circular cone stump, that is, a circular cone chopped off at the top whose base disk has radius $R > 0$, whose height is $h > 0$, and whose top disk has radius $0 < r < R$. Draw a picture first how to view the cone stump as a solid of revolution.

**Problem 3.** Calculate the surface area and volume of the spindle formed by revolving $y = \sin x$ , $x \in [0, 2\pi]$, around the $x$-axis.

**Problem 4.** How much water does the following clay bowl hold? The bowl is made on a pottery wheel and has the shape of revolving the parabola $y = x^2 - 1$ around the $y$-axis,. The bottom of the bowl is the disk at $y = 0$ and the bowl has height $h > 0$ to its top. Draw a picture before calculating. The length unit is inches.

**Problem 5.** A "cooling tower" of height $2h > 0$ is obtained by revolving the hyperbola $y^2 - x^2 = 1$ around the $y$-axis between $-h \leq y \leq h$. Draw a picture. Calculate the amount of concrete needed to build the shell of the cooling tower (assume it infinitely thin). How much volume does it enclose?

**Problem 6.** Consider the curve $\gamma(t) = \cos(2t)(\cos t, \sin t)$ for $t \in [0, 2\pi]$. Draw an accurate picture of this curve and label the points on the curve for $t = \frac{n\pi}{4}$, $n = 0, \ldots, 8$. What is the length of this curve?

**Problem 7.** Consider the integral $\int x^\alpha dx$ where $\alpha \in \mathbb{R}$ is an arbitrary real number. Draw a picture of the graphs of $y = x^\alpha$ for various choices (positive and negative) of $\alpha$ to get an idea of the areas you are supposed to calculate. Answer the following questions:

(i) Find all values of $\alpha \in \mathbb{R}$ so that $\int_a^1 x^\alpha dx$, where $0 < a < 1$, tends to a finite number when $a \to 0$?

(ii) Find all values of $\alpha \in \mathbb{R}$ so that $\int_1^b x^\alpha dx$, where $1 < b < \infty$, tends to a finite number when $b \to \infty$?

(iii) From your above investigations, what can you say about the area under the graphs $y = \frac{1}{x^2}$ between $5 \leq x \leq \infty$? What about $y = \frac{1}{\sqrt{x}}$ between $0 \leq x \leq 4$?

(iv) Provide an argument why the integral

$$\int_1^\infty \frac{\sin(x)}{x^2}\, dx$$

gives a finite value, even though we integrate over an infinitely long interval....This integral is not expressible by elementary functions, so you will have to do some kind of "comparison to something known" argument.

For instance, convince yourselves (how?) that if $0 \leq f(x) \leq g(x)$ then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.