# Homework 3, Honors Calculus II
due 9/27/18

*Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.*

**Problem 1.** An ellipse, centered at the origin, is given by the equation

$$(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$$

where $a \geq b > 0$ are the half axes length. For instance, if $a = b$ then the ellipse becomes a circle of radius $a$. Calculate the area enclosed by the ellipse. In particular, this gives a formula for the area enclosed by a circle.

**Problem 2.** Find the area under of the domain bounded by the parabola $y = x^2 - 1$ and the line passing through the points $(-2, 3)$ and $(3, 8)$. Draw a picture of the region to scale and with labels before attempting the calculation. Also keep in mind that the integral calculates "signed" areas, that is, regions below the $x$-axis have a negative integral—but areas are always positiv.

**Problem 3.** Calculate the area bounded between the graphs of $y = \sin x$ and $y = \cos x$ between their first and second intersection points on the positive $x$-axis. Again, draw a picture of the region first.

**Problem 4.** Find an anti-derivative of each of the following functions:

(i) $f(x) = \cos^3(x)$
(ii) $f(x) = \frac{\ln(x)}{x}$
(iii) $f(x) = x^2 e^x$
(iv) $f(x) = (\sin x)e^{4\cos x}$
(v) $f(x) = \tan x$
(vi) $f(x) = \frac{1}{\sqrt{1+x^2}}$

**Problem 5.** Consider the hyperbola $x^2 - y^2 = 1$. Calculate the area bounded by the two branches of the hyperbola and the two horizontal lines $y = \pm 1$.

**Problem 6.** Find an anti-derivative of

$$f(x) = \frac{1}{x^2 - a^2}$$

for arbitrary $a \geq 0$. This is the first instance of the more general "partial fraction decomposition", which is found in letter exchanges between Bernoulli and Leibniz around 1700.

**Problem 7.** Show that

$$\int_0^{2\pi} \cos(nx)\cos(mx)\,dx = \begin{cases} 0 & \text{if } n \neq m \\ \pi & \text{if } n = m \neq 0 \end{cases}$$

where $n, m \in \mathbb{Z}$ are integers.

**Problem 8.** A function $f\colon [-a, a] \to \mathbb{R}$ is called *even*, respectively *odd*, if $f(-x) = f(x)$, respectively $f(-x) = -f(x)$, for all $x \in [-a, a]$.

(i) Give two examples of an even and two examples of an odd function.
(ii) Show that for an even (continuous) function $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

(iii) Show that for an odd (continuous) function $\int_{-a}^{a} f(x)\,dx = 0$.

(iv) Give an example of a function which is *not* even, but for which (ii) holds.
Give an example of a function which is *not* odd, but for which (iii) holds.