Homework 1, Advanced Calculus
due 2/8/17

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Write down the coordinates in $\mathbb{R}^3$ of the 4 vertices of a tetrahedron (Platonic solid with 4 faces, 6 edges, and 4 vertices; look it up if you have never seen one). For each edge calculate the angle between the two faces sharing this edge (by calculating the angle between the normals to the two faces).

Problem 2. What are the level sets $\{x \in \mathbb{R}^n\,;\, f(x) = c\}$ for the following functions (describe them if they have names and draw an accurate picture indicating the range of c):

(i) $f(x_1, x_2) = (x_1 - x_2)^2$
(ii) $f(x_1, x_2) = x_1^3 - x_2$
(iii) $f(x_1, x_2, x_3) = x_1^2 + x_2^2$
(iv) $f(x_1, x_2) = x_1 x_2$
(v) $f(x_1, x_2, x_3) = x_1^2 - x_2^2 - x_3$

Problem 3. Describe and draw an accurate picture of the graphs of the following functions over the given domains A:

(i) $f(x_1, x_2) = x_1^2 - x_2^2$, $A = \mathbb{R}^2$
(ii) $f(x_1, x_2) = x_1 + x_2$, $A = \mathbb{R}^2$
(iii) $f(x_1) = |x_1|$, $A = [-1, 1]$
(iv) $f(x_1, x_2) = x_1^3$, $A = [-1, 1] \times [-1, 1]$
(v) $f(x_1, x_2) = \cos(x_1^2 + x_2^2)$, A the closed disk of radius $\sqrt{\pi/2}$.

Problem 4. Determine the boundaries of the following sets and give a reason for your answer:

(i) $\{x \in \mathbb{R}^2\,;\, |x| < 1, |y| \leq 1\}$
(ii) $\mathbb{R}^2 \setminus \bar{B}_1(0)$
(iii) $\{(x_1, x_2, x_3) \in \mathbb{R}^3\,;\, x_3^2 > 1\}$
(iv) the subset $\mathbb{Z}$ of integer numbers in $\mathbb{R}$
(v) the subset of rational numbers $\mathbb{Q}$ in $\mathbb{R}$ (Hint: every interval around a rational number contains an irrational number, why?)

Problem 5. Determine whether the following functions are continuous or not, and give a reason for your answer:

(i) $f(x_1, x_2, x_3) = (x_1 x_2 x_3, x_1 + x_2 + x_3)$ on the domain $\mathbb{R}^3$
(ii) the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x_1, x_2) = x_1^2 + x_2^2$ for $x \in B_1(0)$ and $f(x_1, x_2) = 1$ on $\mathbb{R}^2 \setminus B_1(0)$
(iii) the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x_1, x_2) = \frac{1}{x_1 - x_2}$ for $x_1 \neq x_2$ and $f(x_1, x_2) = 0$ for $x_1 = x_2$
(iv) $f(x_1, x_2, x_3) = (\sin(x_1), \cos(x_2), e^{x_3})$ for $x \in \mathbb{R}^3$
(v) the function $f\colon \mathbb{R}^n \to \mathbb{R}$ given by $f(x) = \ln ||x||$ for $||x|| \geq 1$ and $f(x) = 0$ for $||x|| < 1$.