Homework 1, Differential Geometry
due 2/3/17

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Give a parameterization of the once, in counterclockwise direction, traversed ellipse centered at the origin with major and minor axis length $2a$ and $2b$ respectively.

Problem 2. Give a parameterization of the right branch of the hyperbola $x^2 - y^2 = 1$. Can you find a unit speed parametrization?

Problem 3. Give an arclength parameterization of the twice, in counterclockwise direction, traversed circle centered at the origin of radius r.

Problem 4. Reparametrize the helix $\gamma\colon \mathbb{R} \to \mathbb{R}^3$, $\gamma(t) = (\cos t, \sin t, t)$ so that the reparametrized curve $\tilde{\gamma} = \gamma \circ \varphi$ has speed 1.

Problem 5. Complete the proof from class, namely that the length of a curve is paramterization invariant, for the case when the reparametrization is orientation reversing. Go over each step carefully and provide all details.

Problem 6. Consider the tractrix curve $\gamma\colon (0, \pi) \to \mathbb{R}^2$ given by

$$\gamma(t) = (\sin(t), \cos(t) + \ln \tan(t/2))$$

Verify that from any point on the curve the line segment of the tangent line from the curve point to the y-axis has length 1. Draw a picture of the curve and those line segments for various points on the curve. Can you give an example from real life where such a curve might appear?

Problem 7. Let $\gamma\colon [a, b] \to \mathbb{R}^n$ be a smooth curve and $a = t_0 < t_1 < \cdots < t_n = b$ an equidistant partition of $[a, b]$, i.e. $t_{k+1} - t_k = 1/n$ for all $k = 0, \ldots, n-1$. Consider the (open) polygon P_n with vertices $\gamma(t_0), \gamma(t_1), \ldots, \gamma(t_n)$ and denote its length by $L(P_n)$. Show that

$$\lim_{n\to\infty} L(P_n) = L(\gamma) = \int_a^b ||\gamma'(t)||dt$$