Homework 1, Honors Calculus II
due 9/11/18

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. In class we defined the length $L(\overline{AB})$ of a line segment with endpoints $A, B \in \mathbb{R}^2$ in the plane. Provide a mathematical proof/argument/calculation of the following statements:

(i) L is translation invariant, i.e., if you translate the line segment by some vector $T \in \mathbb{R}^2$, then its length does not change.

(ii) L is rotation invariant, i.e., if you rotate the line segment by some angle α around some point in the plane, then its length does not change.

Here some thoughts/hints to item (ii): once you know that L is translation invariant, you can (without loss of generality, as mathematicians say, meaning that using item (i) you can reduce to this case) safely assume that the segment is rotated around one of the endpoints of the line segment which you could even assume to be the origin (why?). Now find a formula for the coordinates of the point $(\tilde{x}, \tilde{y}) \in \mathbb{R}^2$ obtained by rotating by angle α in anti-clockwise direction a point $(x, y) \in \mathbb{R}^2$. So you aim for a formula

$$\tilde{x} = \text{ some expression in } x \text{ and } \alpha$$

and the same for $\tilde{y}$. If you already know some vector/matrix notation, you could apply this here. If not, you just find the formula. Draw pictures!

Problem 2. Take a circle of radius $R > 0$. Try to calculate its length by inscribing regular n-gons for larger and larger $n = 3, 4, 5, \ldots$.

Problem 3. In the previous example, you could also surround (have no better word for it, just draw a picture) the circle by regular n-gons, so that the circle gets inscribed. Intuitively it is clear that the surrounding n-gons give a larger value for the (approximative) circumference of the circle, and the inscribed n-gons a smaller value. The Greeks called the ratio (which they believed should exist)

$$\frac{\text{circumference}}{2R} =: \pi\,.$$

Can you arrive at some reasonable value of π using the polygonal approximation? Try a regular inscribed and surrounded 3-gon, then do the same with a square, perhaps also a regular 6 and 8-gon to get a feel about the difference between the outside and inside approximation...compare how good/bad this is to the value of π your calculator produces.