Homework 5: Due Nov 28th, 2018

1. Consider the symmetric matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

   Let us try to find the eigenvalue nearest $\alpha = 1$ by hand. Do two steps of the shifted inverse iteration matrix on A with $\alpha = 1$ starting from $\mathbf{x}^0 = [1, 1, 1]^T$. Compute the Rayleigh quotient on $\mathbf{x}^2$. Compare this to the true eigenvalue, found numerically.

2. (For this problem, you do not need to show work by hand.) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

   using the QR iteration. Use the internal QR decomposition solver `qr()` for each iteration. For five iterations, compute the difference between the diagonal $\texttt{diag}(A_k)$ and the spectrum $\texttt{eig}(A)$ (note: you should sort both of the vectors using `sort`). Give the error in a table for each of the eigenvalues (so thirty values total). Do some seem to be converging faster than others?

3. Find all symmetric 2 by 2 matrices so that the QR iteration converges in a single step. That is find all

$$A_0 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

   so that A_1 is diagonal.

4. Use the Gram-Schmidt process to compute the QR decomposition of the matrix

$$A = \begin{bmatrix} 1 & 10 & 10 \\ 1 & 4 & -4 \\ -1 & 4 & 6 \\ -1 & -2 & -8 \end{bmatrix}$$