

Homework 4: Due Nov 9th, 2018

1. For the conjugate gradient method, prove the following.

- (a) $\text{span}\{\mathbf{p}_0, \mathbf{p}_1, \dots, \mathbf{p}_{k-1}\} = \text{span}\{\mathbf{r}_0, \mathbf{r}_1, \dots, \mathbf{r}_{k-1}\} = \text{span}\{\mathbf{r}_0, A\mathbf{r}_0, \dots, A^{k-1}\mathbf{r}_0\}$
- (b) $\langle \mathbf{p}_k, A\mathbf{p}_\ell \rangle = 0$ for all $\ell < k$.
 $\langle \mathbf{r}_k, \mathbf{r}_\ell \rangle = 0$ for all $\ell < k$.
- (c)

$$\alpha_k = \frac{\langle \mathbf{p}_k, \mathbf{r}_k \rangle}{\langle \mathbf{p}_k, A\mathbf{p}_k \rangle} = \frac{\langle \mathbf{r}_k, \mathbf{r}_k \rangle}{\langle \mathbf{p}_k, A\mathbf{p}_k \rangle}$$
$$\frac{\langle -\mathbf{r}_{k+1}, A\mathbf{p}_k \rangle}{\langle \mathbf{p}_k, A\mathbf{p}_k \rangle} = \frac{\langle \mathbf{r}_{k+1}, \mathbf{r}_{k+1} \rangle}{\langle \mathbf{r}_k, \mathbf{r}_k \rangle}$$

Use induction on k , proving the three parts together.

2. Problem 7.7.25 from the text. Use both CG and multigrid methods. You do not need to use preconditioning.