Homework 2: Due Feb. 10th, 2016
All answers must be justified.

1. Let $V = C^1([0, 1]; \mathbb{R})$ denote the vector space of continuous functions whose derivatives are continuous functions on the closed interval $[0, 1]$. Prove whether or not the following defines an inner product on $V$:

$$\langle f, g \rangle = \int_0^1 f'(x)g'(x) \, dx.$$ 

2. For the vector space $\mathbb{R}^n$, prove whether or not the following defines an inner product:

$$\langle v, w \rangle = \sum_{j=1}^n j^2 v_j w_j.$$ 

3. Given an inner product space $V$ with the induced norm $\|v\| := \langle v, v \rangle^{1/2}$, prove the parallelogram law:

For all $v, w \in V$, $2\|v\|^2 + 2\|w\|^2 = \|v + w\|^2 + \|v - w\|^2$.

Draw a parallelogram with sides given by vectors $v$ and $w$ to illustrate the theorem.

4. Use Gram-Schmidt to compute the QR decomposition of the matrix

$$A = \begin{bmatrix}
1 & 10 & 10 \\
1 & 4 & -4 \\
-1 & 4 & 6 \\
-1 & -2 & -8
\end{bmatrix}$$

5. Use Gram-Schmidt to produce an orthonormal basis from the basis $\beta = (x^2, 1, x)$, with inner product $\langle f, g \rangle = \int_{-1}^1 f(x)g(x) \, dx$. Note: the nonstandard ordering is intentional.