Solutions for Practice Midterm 1

Alexei Oblomkov

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Note: these are answers for the question, but for full credit on the exam, you must show your work and justify your answers.

Problem 1

a)

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b) Not invertible, the echelon form does not have a pivot in every row.
c) The (1,1), (2,2), and (3,4) entries
d) Having no solution is possible. Having infinitely many solutions is possible. Having a one unique solution is not possible.

Problem 2

a)

$$\mathbf{x} = \begin{bmatrix} 1-s \\ 2+2s \\ s \end{bmatrix}$$

b)

$$\mathbf{x} = \begin{bmatrix} -s \\ 2s \\ s \end{bmatrix}$$

c) No because the echelon form of A has a row of zeroes.

Problem 3.

The echelon form of the matrix $[v_1, v_2, v_3]$ is

$$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{bmatrix}.$$

Hence

a) the vectors v_1, v_2, v_3 are linearly independent because there is a pivot in every column

b) they span $\mathbb{R}^3$ because there is a pivot in every row.

Problem 4.

a)The matrix of the linear transform is

$$\begin{bmatrix} 0 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 \end{bmatrix}$$

It is neither one-to-one nor is onto.

b) The matrix of the linear transform is

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \\ 7 & 1 \end{bmatrix}$$

Since the echelon form of this matrix is

$$\begin{bmatrix} 1 & * \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

this linear transform is one-to-one but not onto.

Problem 5.

a)

$$\begin{bmatrix} 8 & -3 & 1 \\ -17 & 7 & -3 \\ 5 & -2 & 1 \end{bmatrix}$$

b) The solution is $A^{-1}v$ where

$$v = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}.$$

The solution is

$$\begin{bmatrix} 12 \\ -28 \\ 9 \end{bmatrix}.$$

Problem 6.

a)$2/3A - 1/3C^{-1}B^{-1}FE^{-1}D^{-1}$. The solution is unique.
b) If A or F were singular, the solution above is still valid and unique. However, if B, C, D, E are singular, then there isn't a solution for all F.