Math 235 Practice Final.

Instructions: Exam time is 2 hours. You are allowed one sheet of notes (letter-size paper, both sides). Calculators, the textbook, and additional notes are *not* allowed. Justify all your answers carefully.

(1) Write the matrix in the standard basis for the linear transformation T such that

$$T(\begin{bmatrix} 1 \\ 3 \end{bmatrix}) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}.$$

(2) Compute determinant of the matrix:

$$\begin{bmatrix} 1 & -1 & 2 & -2 \\ -1 & 2 & 1 & 6 \\ 2 & 1 & 14 & 10 \\ -2 & 6 & 10 & 33 \end{bmatrix}.$$

(3) Let x be a real number and

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Find the values of x such that A can be diagonalized as $A = PDP^{-1}$ and give the corresponding matrices P and D. For any value of x such that A cannot be diagonalized, explain what goes wrong.

(4) Find the complex eigenvalues and associated eigenvectors for the matrix

$$A = \begin{pmatrix} 1 & -37 \\ 1 & 13 \end{pmatrix}$$

(5) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 0 & 2 & -1 \\ -1 & 2 & 0 \\ -1 & 2 & 0 \end{bmatrix}$$

(6) (a) Find a basis of the null space and a basis of the column space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

(b) What are the eigenvalues of A? (Hint: Col$A$ is the range of the linear map $\mathbf{x} \mapsto A\mathbf{x}$.)

(c) Diagonalize A.

(7) Give, with explanation, the maximal and minimal possible ranks (which may be equal) of the following matrices:

(a) a 5×7 matrix

(b) a 3×5 matrix with a row of zeroes.

(c) a 4×4 matrix whose rows all sum to zero.

(8) Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} \frac{1}{10} \\ \frac{-1}{10} \\ \frac{7}{10} \\ \frac{-7}{10} \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} \frac{7}{10} \\ \frac{-7}{10} \\ \frac{-1}{10} \\ \frac{1}{10} \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix},$$

Is the set $\{v_1, v_2, v_3\}$ orthonormal? Justify your response.

Compute $\text{proj}_L \mathbf{e}_1$ where L is the line spanned by $\mathbf{v}_1$ and $\mathbf{e}_1$ is the first standard unit vector.

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