Math 612 Midterm Practice

Midterm Exam will be on Thursday, 3/14, in class

Midterm policies:

- The midterm will cover chapter 13 and sections 14.1 - 14.3 of Dummit and Foote.
- The midterm will consist of 5-7 problems that vary in difficultly. The problems will be similar to homework problems, problems assigned in lecture, or old qualifying exam problems.
- You may bring a single-sided 8.5 × 11 inch handwritten notesheet (although this is not required). There are no restrictions on what can be written on the sheet.
- Exams will be graded and returned through Gradescope.

Midterm style problems.

1. Let $\mathbb{F}$ be a finite field of characteristic p. Prove that $|\mathbb{F}| = p^n$ for some $n > 0$.
2. Prove that $x^3 - 2$ is irreducible over $\mathbb{Q}(i)$.
3. Let p_1, p_2 be distinct primes. Show that $\mathbb{Q}(\sqrt{p_1}, \sqrt{p_2}) = \mathbb{Q}(\sqrt{p_1} + \sqrt{p_2})$.
4. Determine the splitting field and its degree over $\mathbb{Q}$ for $x^6 - 4$.
5. Let $\phi : \mathbb{F} \to \mathbb{F}$ be the Frobenius map $\phi(\alpha) = \alpha^p$ where $\mathbb{F}$ is a finite field of characteristic p. Prove that ϕ^n is the identity and no lower power of ϕ is the identity.
6. For any prime p and nonzero $a \in \mathbb{F}_p$, prove that $x^p - x + a$ is irreducible and separable over $\mathbb{F}_p$.
7. Determine the automorphisms of $\mathbb{Q}(\sqrt[4]{2})$ over $\mathbb{Q}$.
8. Let K/F and K'/F be field extensions. Let $\phi : K \to K'$ be an isomorphism of $K \cong K'$ over F. Prove that there is a group isomorphism $\mathrm{Aut}(K/F) \cong \mathrm{Aut}(K'/F)$ given by $\sigma \mapsto \phi\sigma\phi^{-1}$.
9. Determine the Galois group of $x^5 - 5$ over $\mathbb{Q}$.
10. Determine the minimal polynomial over $\mathbb{Q}$ of $1 + \sqrt[3]{2} + \sqrt[3]{4}$.
11. Suppose K is a Galois extension of F of degree p^n for some prime p and $n > 0$. Show that there are Galois extensions of F contained in K of degrees p and p^{n-1}.
12. Prove that is the Galois group of the splitting field of a cubic over $\mathbb{Q}$ is cyclic of order 3, then all roots of the cubic are real.