Math 611 Homework 9

Due Friday, December 1, 2023 to Gradescope (by 11:59 pm)

The problem numbers below refer to Dummit and Foote, third edition.

Homework policies:

1. Homeworks will vary in length from 10 - 20 problems, depending on length and difficulty of the problems. A subset of the problems will be graded for correctness.
2. You can neatly handwrite or type your homework, and do not need to copy the problem statement. Please clearly label each problem with its number/part.
3. You may use any result from a previous section of the textbook or previous homework assignment. Please indicate that you have done so (e.g. 'by Proposition 2 in §1.1, part (2) ... ' or 'by Homework 2, Problem 4...').
4. If you collaborate with others, please write their names at the top of your assignment.
5. For most homework assignments, I will include 1 - 2 sample qualifying exam problems related to the content of the assignment. You *do not* have to complete these problems or turn them in, but they are good indications of your mastery of the material.

Assigned problems:

- §8.1: 8a, 9, 10
- §8.2: 1, 3, 5, 6bc (you may take 6(a) as given because we have not discussed Zorn's Lemma, i.e. you may assume that if the set of non-principal ideals in R is not empty, then it has a maximal element)
- §8.3: 5, 8
- §10.1: 5, 6, 8, 9
- §10.2: 3, 9, 13

Sample qualifying problem related to this section:

Fall 2019 Exam, Problem 4:
Prove that every prime ideal in $\mathbb{Z}[x]$ can be generated by at most two elements.

Fall 2019 Exam, Problem 6:
Let R denote the ring $Z[\sqrt{-5}]$. Let p be the ideal $(3, 1+\sqrt{-5})$ in R.

1. Show that p is a prime ideal.
2. Show that p is not a principal ideal.
3. Let S be the complement of p in R. Show that the ideal $S^{-1}p$ is principal in the localization $S^{-1}R$.