Math 611 Homework 4

Due Friday, October 13, 2023 to Gradescope (by 11:59 pm)

The problem numbers below refer to Dummit and Foote, third edition.

Homework policies:

1. Homeworks will vary in length from 10 - 20 problems, depending on length and difficulty of the problems. A subset of the problems will be graded for correctness.
2. You can neatly handwrite or type your homework, and do not need to copy the problem statement. Please clearly label each problem with its number/part.
3. You may use any result from a previous section of the textbook or previous homework assignment. Please indicate that you have done so (e.g. 'by Proposition 2 in §1.1, part (2) ... ' or 'by Homework 2, Problem 4...').
4. If you collaborate with others, please write their names at the top of your assignment.
5. For most homework assignments, I will include 1 - 2 sample qualifying exam problems related to the content of the assignment. You *do not* have to complete these problems or turn them in, but they are good indications of your mastery of the material.

Assigned problems:

- §3.2: 4, 5, 16
- §3.3: 2, 4
- §3.4: 2, 5, 7
- §3.5: 5, 15, 16, 17
- §4.1: 4, 10

Sample qualifying problem related to this section:

Fall 2022 Exam, Problem 2(c):

First, a reminder that 2(b) was the following (from Homework 2): let p be a prime and let H be the subset of upper triangular matrices in $GL_2(\mathbb{F}_p)$ with 1's on the diagonal. Prove that the center of H is

$$Z(H) = \left\{ \begin{bmatrix} 1 & 0 & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mid c \in \mathbb{F}_p \right\}.$$

Then, 2(c) is: show that $H/Z(H) \cong \mathbb{Z}_p \times \mathbb{Z}_p$.

Fall 2020 Exam, Problem 2: Let G be a finite group acting on a finite set X. For any $g \in G$, let $X_g := \{x \in X \mid g \cdot x = x\}$ be the *fixed point set* of g. Prove that the number of G-orbits in X is

$$\frac{1}{|G|} \sum_{g \in G} |X_g|.$$

Hint: first prove that

$$\sum_{x \in X} |G_x| = \sum_{g \in G} |X_g|$$

where G_x is the stabilizer of $x \in X$.