Math 611 Homework 3

Due TUESDAY October 3, 2023 to Gradescope (by 11:59 pm)

Note: this homework is slightly longer than previous ones, but you have more time to complete it! I will have office hours on Monday and Tuesday, October 2 and 3.

The problem numbers below refer to Dummit and Foote, third edition.

Homework policies:

1. Homeworks will vary in length from 10 - 20 problems, depending on length and difficulty of the problems. A subset of the problems will be graded for correctness.
2. You can neatly handwrite or type your homework, and do not need to copy the problem statement. Please clearly label each problem with its number/part.
3. You may use any result from a previous section of the textbook or previous homework assignment. Please indicate that you have done so (e.g. 'by Proposition 2 in §1.1, part (2) ... ' or 'by Homework 2, Problem 4...').
4. If you collaborate with others, please write their names at the top of your assignment.
5. For most homework assignments, I will include 1 - 2 sample qualifying exam problems related to the content of the assignment. You *do not* have to complete these problems or turn them in, but they are good indications of your mastery of the material.

Assigned problems:

- §2.3: 6 (please use '. . . ' as needed for problem 6), 11, 15, 24, 26
- §2.4: 10 (reminder for 10: you saw $SL_n(F)$ in Homework 2, exercise 2.1.9) 15, 19
- §2.5: 5
- §3.1: 1, 4, 5, 9, 11, 12, 18, 34, 36

Sample qualifying problem related to this section:

Spring 2023 Exam, Problem 1:

(a) Let G be a group, and let H be a subgroup of finite index. Show that G has a normal subgroup N with finite index such that $N \leq H$.

(b) Let G be a group, and let H_1, H_2 be subgroups of finite index. Show that $H_1 \cap H_2$ also has finite index in G.