Math 411 Homework 3

Due March 3, 2024 to Gradescope (by midnight)

The problem numbers below refer to Saracino's text, second edition. This homework covers Section 4 and 5. You can neatly handwrite or type your homework, and do not need to copy the problem statement, but please clearly label each problem that you are working on. If you collaborate with others, please write their names at the top of your assignment.

1. Problem 4.1: For $\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, list the elements of: $\langle 0 \rangle$, $\langle 1 \rangle$, $\langle 2 \rangle$, $\langle 3 \rangle$, $\langle 4 \rangle$, $\langle 5 \rangle$, and $\langle 8 \rangle$.
2. Problem 4.11: Is $GL(2, \mathbb{R})$ cyclic?
3. Problem 4.17: Prove that if $G = \langle x \rangle$ and G is infinite, then x and x^{-1} are the only generators of G.
4. Problem 4.19: Prove that the order of x is equal to the order of x^{-1}.
5. Problem 4.21: Show that for any two elements $x, y \in G$, $o(xy) = o(yx)$.
6. Problem 4.22: Let G be an abelian group and let $x, y \in G$. Suppose that x, y have finite order. Show that xy has finite order and $o(xy)$ divides $o(x)o(y)$.
7. Problem 5.3: Let H be the set of matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in $G = GL_2(\mathbb{R})$ such that $ad - bc = 1$. Show that H is a subgroup of G.
8. Problem 5.11: Let G be an abelian group and let n be a positive integer. Let H be the subset of G consisting of all $x \in G$ such that $x^n = e$. Show that H is a subgroup of G.
9. Problem 5.22: Let G be a group. Prove that $Z(G)$ is a subgroup of G.
10. Problem 5.25: Let G be a group and let a be some fixed element of G. Let H be a subgroup of G and let aHa^{-1} be the subset of G consisting of all elements of the form aha^{-1}, with $h \in H$. Show that aHa^{-1} is a subgroup of G.