Math 411 Homework 1

Due February 11, 2022 to Gradescope (by 11:59 pm)

The problem numbers below refer to Saracino's text, second edition. This homework covers Section 1 and the first part of Section 2. You can neatly handwrite or type your homework, and do not need to copy the problem statement, but please clearly label each problem that you are working on. If you collaborate with others, please write their names at the top of your assignment.

1. Problem 1.3, parts (e), (f), (h): For each set and operation below, determine whether or not $\star$ is a binary operation on S.
   (a) $S = \mathbb{Z}$, $a \star b = a + b - ab$
   (b) $S = \mathbb{R}$, $a \star b = b$
   (c) $S = \{1, 6, 3, 2, 18\}$, $a \star b = ab$

2. Problem 1.6, parts (e), (f), (h): For each part in the previous problem where $\star$ *is* a binary operation, determine if $\star$ is commutative and/or associative.

3. Problem 1.9: If S is a finite set, we define a binary operation on S by writing down all the values of $s_i \star s_j$ in a table. If $S = \{a, b, c, d\}$, the following gives a binary operation on S.

$\star$	a	b	c	d
a	a	c	b	d
b	c	a	d	b
c	b	d	a	c
d	d	b	c	a

   Here, $s_1 \star s_2$ is the element in row s_1 and column s_2. For example, $c * b = d$. Is the above binary operation commutative? Is it associative?

4. Problem 2.1, parts (b), (c), (d), (e), (i): Which of the following are groups? Why?
   (a) The set $3\mathbb{Z}$ of integers that are multiples of 3, under addition.
   (b) $\mathbb{R} - \{0\}$ under the operation $a \star b = |ab|$.
   (c) The set $\{1, -1\}$ under multiplication.
   (d) The subset of $\mathbb{Q}$ of positive rational numbers with rational square roots, under multiplication.
   (e) $\mathbb{Z}$, under the operation $a \star b = a + b - 1$.

5. Problem 2.4: Write down the tables for the binary operation on the following groups:
   (a) $(\mathbb{Z}_4, + \pmod 4)$
   (b) $(\mathbb{Z}_5, + \pmod 5)$
   (c) $(\mathbb{Z}_6, + \pmod 6)$

6. Problem 2.5: The following table defines a binary operation on $S = \{a, b, c\}$. Is $(S, \star)$ a group?

*	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

7. (a) Problem 2.7: Let $S = \{a, b\}$. Write down a table that defines a binary operation $\star$ on S such that $(S, \star)$ is a group. Show that or explain why your table works.
   (b) Is your table unique? i.e. can any other table give a binary operation that makes $(S, \star)$ into a group?