

Homework 10/ Practice Midterm 2

LIUBOMIR CHIRIAC

1. (a) Show that $25 \mid 2^{65} + 3^{65}$. (Hint: Use Euler's Theorem.)
(b) Let $p > 3$ be prime. Find the remainder when $3^p(p-2)!$ is divided by p .
(Hint: Combine Wilson's Theorem and Fermat's Little Theorem.)
2. Suppose that both p and $2p-1$ are odd primes. Let $n = 2(2p-1)$. Prove that

$$\varphi(n) = \varphi(n+2).$$

(Hint: Use the multiplicative property of φ , i.e., Lemma 9.2.8 in your textbook.)

3. Suppose the RSA algorithm is used with the modulus $n = 91$.
 - (a) List four possible values for the encryption exponent e .
 - (b) Let $e = 17$. First, encrypt the message 10 and then encrypt again the answer you obtained.
 - (c) Based on your computations above, explain why the choice made for e in part (b) may not be considered too secure.
4. (a) Show that the order of any nonzero element in \mathbb{Z}_{23} is either 1, 2, 11 or 22.
(b) Show that 5 is a primitive root modulo 23. (Hint: Use part (a).)
(c) Part (b) implies that every nonzero element of \mathbb{Z}_{23} appears exactly once in the list

$$\bar{5}, \bar{5}^2, \dots, \bar{5}^{22}.$$

Find all the elements in this list which are primitive roots in \mathbb{Z}_{23} .

- (d) Find the order of 5^{14} modulo 23.