46) Let $C$ denote the circle of radius 2 centered at the origin, oriented counter-clockwise. Compute:

(a) $\int_C \coth z \, dz$;  
(b) $\int_C \frac{e^{\pi z}}{z(z^2 + 1)} \, dz$;  
(c) $\int_C \frac{1}{z(1 - z^2)} \, dz$

47) Let $f(z) = \frac{1}{(z^2 - 3z + 2)}$. Find the Laurent series for $f$:

a) In the disk $|z| < 1$.

b) In the annulus $1 < |z| < 2$.

c) In the region $|z| > 2$.

48) Find the Laurent expansion of $f(z) = \frac{1}{(z - 1)^2(z + 1)}$ valid in the annulus $\{0 < |z - 1| < 2\}$.

49) Compute the first four terms of the Laurent expansion of the function $f(z) = \frac{e^z}{z(z^2 + 1)}$ valid in the region $0 < |z| < 1$.

50) a) Give a careful statement and proof (using complex analysis) of the Partial Fraction Decomposition Theorem for rational functions over $\mathbb{C}$.

b) Use complex analysis to obtain the partial fraction decomposition of the rational function:

$$\frac{z + 1}{z^3(z^2 + 1)^2}$$

51) Problem 4, page 103.

52) Problem 5, page 103.

53) Problem 6, page 104.

54) Problem 8, page 104.

55) Problem 12, page 105.